1. (15 points) Derive a formula for the Newton's method for the equation

$$\frac{f(x)}{f'(x)} = 0.$$

Show that the convergence of the method to any zero of $f(x)$ is of second order, even if the zero is not simple.

2. (15 points) Consider the equation

$$f(x) = \sin x - x^3 - 1 = g(x) - h(x)$$

where $g(x) = \sin x$ and $h(x) = x^3 + 1$. Which of the following two methods will converge to the root near $x = -1$: a) $h(x_{n+1}) = g(x_n)$, b) $g(x_{n+1}) = h(x_n)$?

3. (15 points) Use $f(x)$, $f(x+h)$, $f(x+2h)$, and the method of undetermined coefficients to approximate $f'(x)$.

4. (15 points) Suppose that the experimental data (x_i, y_i), $i = 1, 2, \cdots, n$, behave like $y = \alpha \exp(\beta x)$. Derive a least-square method based on this assumption.

5. (15 points) Find a rational function of the form $r(x) = (ax + b)/(cx + d)$ to approximate the function $f(x) = \sqrt{1+x}$, $0 \le x \le 1$, where a, b, c, and d are constants to be determined.

6. (25 points) Consider

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 6 & 2 \\ -1 & 1 & 3 \end{bmatrix}.$$

Use Gauss elimination in exact arithmetic with scaled row pivoting to obtain the factorization

$$PA = LDU$$

where P is a permutation matrix, L is a unit lower triangular matrix, U is a unit upper triangular matrix, D is a diagonal matrix.

試題隨卷繳回