國立台灣大學九十二學年度碩士班招生考試試題

科目：應用數學(C)

1. [**3 pt.**] (a) Find the eigenvalues λ_1, λ_2 and normalized eigenvectors $\mathbf{e}_1$, $\mathbf{e}_2$ of $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 4 & 3 \end{pmatrix}$.

   [**3 pt.**] (b) For $\mathbf{v}^T = (1,\ 2)$, find α_1, α_2 such that $\mathbf{v} = \alpha_1\mathbf{e}_1 + \alpha_2\mathbf{e}_2$.

   [**9 pt.**] (c) Find solutions to $\begin{cases} 3x_1 + 4x_2 = \lambda x_1 + 1 \\ 4x_1 + 3x_2 = \lambda x_2 - 1 \end{cases}$, for i) $\lambda = 2$, and ii) $\lambda = \lambda_1$ or λ_2.

2. [**7 pt.**] (a) Use Gauss-Jordan reduction to obtain the solution to $\begin{cases} 3x_1 + 2x_2 + x_3 = 11 \\ 2x_1 + 3x_2 + x_3 = 13 \\ x_1 + x_2 + 4x_3 = 12 \end{cases}$.

   [**6 pt.**] (b) Write the set of linear equations in (a) as $\mathbf{A}\mathbf{x} = \mathbf{c}$. Find $\mathbf{A}^{-1}$ explicitly and use this to solve for $\mathbf{x}$.

3. Let $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \mathbf{A}^5 - 3\mathbf{A}^4 + 2\mathbf{A} - \mathbf{I}$.

   [**8 pt.**] (a) Find the eigenvalues of $\mathbf{B}$ and determine whether $\mathbf{B}$ is positive definite.
   [**4 pt.**] (b) Determine the elements of $\mathbf{A}^{50}$.

4. [**10 pt.**] For $\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$, determine the eigenvalues and eigenvectors of $\mathbf{A}\,\mathbf{x} = \lambda\,\mathbf{B}\,\mathbf{x}$. Verify that the eigenvectors are orthogonal relative to both $\mathbf{A}$ and $\mathbf{B}$.

5. [**10 pt.**] Find the general solution to $x^2y'' + 3xy' - 3y = 0$.

6. [**10 pt.**] Find a particular solution of $y'' + y' - 6y = e^{-x}$ by both the method of undetermined coefficients and the method of variation of parameters.

7. [**3 pt.**] (a) Define analyticity.
   [**3 pt.**] (b) If $f(x) = \Sigma_{n=0}^{\infty} a_n x^n$ exists, show that $a_n = \frac{f^{(n)}(0)}{n!}$.
   [**4 pt.**] (c) Define what is a regular singular point (at $x = 0$) for $y'' + p(x)y' + q(x)y = 0$, and give the indicial equation.

8. [**10 pt.**] Solve $x^2y'' + xy' + x^2y = 0$ for the Bessel function of first kind of order zero by series expansion.

9. [**5 pt.**] (a) For $f(x) = |x|$, $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$, plot $f(x)$ and find its Fourier series.
   [**5 pt.**] (b) Solve $y'' + y = g(t)$ by using Laplace transform, for initial conditions $y(0) = 0$, $y'(0) = 0$, and

$$g(t) = \begin{cases} 0 & t < 3 \\ (t-3)/4 & 3 < t < 7, \\ 1 & t > 7 \end{cases}$$

試題隨卷繳回