

1. Use the energy method to determine the deflection of the following pin-jointed cantilever truss under a loading  $W$ . All the members have a cross-sectional area  $A$  and elastic modulus  $E$ . (25%)

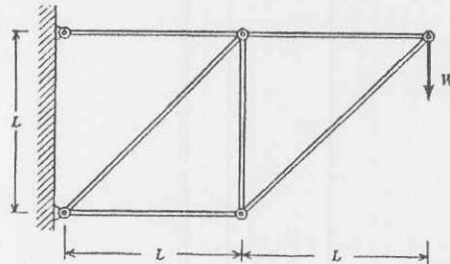


Figure 1

2. A chalk of diameter 12 mm, length 100 mm, was pulled to fail at 5 N. Determine the torque at which the same chalk will fail when subjected to a torsion test. Also describe how the chalk fails both in a tensile and a torsion test and explain why the shapes of the broken cross-section (draw a simple diagram) are the way they are. (25%)
3. (Total 25%) Precision measurements of molecular-beam momentum are made by using a tiny aluminum blade which is built-in at one end and simply supported at the other, as shown in the figure. The blade's length is  $L = 300\text{ mm}$  and its cross section is rectangular,  $0.5 \times 0.01\text{ cm}^2$ . Young's modulus for aluminum is  $E = 68\text{ GN/m}^2$ . The molecular beam impinges on one side of the blade at a distance  $0.5255L$  (for greatest sensitivity) and is equivalent to a force  $P$ . A light beam is reflected from the other side at a distance  $\beta L$  where the beam's angular deflection  $\phi$  is a maximum.
- Derive the formula of the deflection curve of the aluminum blade in terms of force  $P$ . (10%)
  - Find the location  $\beta L$  for the maximum angular deflection. (5%)
  - Find the value of the maximum angular deflection ( $\phi_{\max}$ ). (5%)
  - Find the sensitivity ratio  $k = S/P$ . (5%)

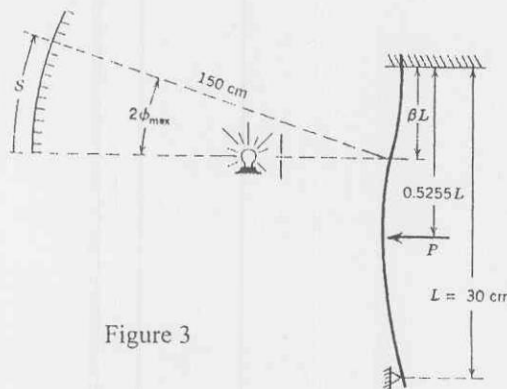


Figure 3

4. (Total 25%) Consider a beam under distributed load  $q(x)$ , as shown in Figure 4(a). By cutting an infinitesimal element  $dx$  at position  $x$  and considering the equilibrium of this small element, as shown in Figure 4(b), we can derive the two well known equations relating the bending moment  $M$ , shear force  $V$ , and  $q(x)$ :

$$\frac{dV}{dx} + q = 0, \quad \frac{dM}{dx} + V = 0. \quad (4.1)$$

- (a) In deriving these two equations we examine the equilibrium condition assuming that the beam remains straight and horizontal even though it is under lateral load  $q(x)$ . We are not satisfied with this ignorance of the beam deflection and decide to rederive the equations by cutting a small element from the deflected beam, as shown in Figure 4(c). What are the two new equations analogous to Eq. (4.1)? Please note that the two sides of the small element are vertical. (5%)
- (b) We consider a slight complication of the above problem by studying the beam under an axial force  $P$  in addition to the distributed lateral load  $q(x)$ , as shown in Figure 4(d). Now re-derive the two equations analogous to Eq. (4.1) by assuming that the beam remains straight and horizontal. (5%)
- (c) Redo problem 4(b) but consider the equilibrium condition of the beam in its deformed configuration. Note that the two sides of the small element are vertical. The free body of the small element is shown in Figure 4(e). (5%)
- (d) Redo problem 4(c), i.e., the equilibrium of the small element of a beam under both lateral force  $q(x)$  and axial force  $P$  in its deformed configuration, but this time the two sides are cut normal to its neutral axis. The free body of the small element is shown in Figure 4(f). (5%) What are the relations between the shear force  $V^*$ , bending moment  $M^*$ , axial force  $P^*$  in Figure 4(f) and their counterparts in Figure 4(e)? (5%)

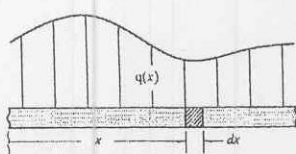


Figure 4(a)

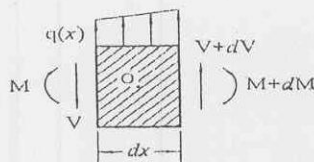


Figure 4(b)

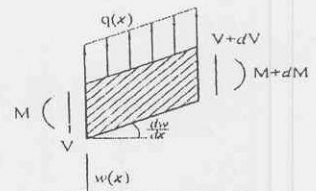


Figure 4(c)

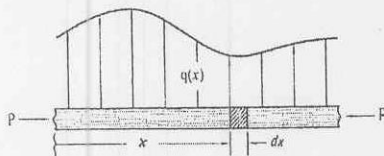


Figure 4(d)

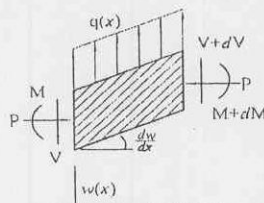


Figure 4(e)

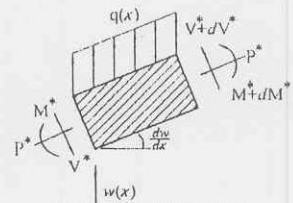


Figure 4(f)