1. Please explain the following statistical terms (Total 30%)

    (1). Unbiased Estimator (3%)

    (2). Minimum Variance Unbiased Estimator (5%)

    (3). Standard Error (3%)

    (4). Central Limit Theorem (5%)

    (5). Type I Error (3%)

    (6). Type II Error (3%)

    (7). Significance Level of a test (4%)

    (8). Statistical Power of a test (4%)

2. Let X be a random variable with p.d.f. $f(x) = \lambda e^{-\lambda x}$, for $x \geq 0$; otherwise $f(x) = 0$, where $\lambda > 0$. (Total 15%)

    (1). Please prove that the total probability of $f(x)$ is 1 (5%).

    (2). Please find the mean and variance of X (5%).

    (3). Please derive the cumulative distribution function (c.d.f.) of X (5%).

3. Let $Y_i \sim$ iid $\mathcal{N}(\mu, \sigma^2)$, i = 1, 2, 3, ..., n (Total 20%)

(1). Let $Z_i = (Y_i - \mu)/\sigma$, what is the distribution of Z_i and what are the mean and variance of Z_i? (2%)

(2). Let $T = \sum_{i=1}^{n} Z_i$, what is the distribution of T and what are the mean and variance of T? (3%)

(3). Let $G = \sum_{i=1}^{n} Z_i^2$, what is the distribution of G and what are the mean and variance of G? (5%)

(4). Let M $= \sum_{i=1}^{n} Y_i \Big/ n$, then what is the distribution of M and what are the mean and variance of M? (5%)

(5). Assuming σ is known, please construct a two-sided 95% confidence interval on μ based on M (5%).

4. Let $Y_i \sim$ iid f, i = 1, 2, 3, ..., n, where f is a proper p.d.f. with a finite mean μ and variance σ^2. As n $\rightarrow \infty$, then (Total 15%)
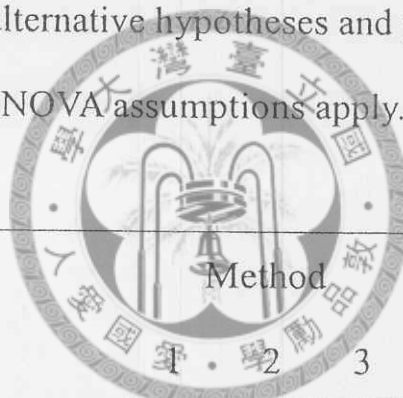
(1). What is the limiting distribution of T, where $T = \sum_{i=1}^{n} Y_i$, and what are the mean and variance of this limiting distribution? (4%)

(2). What is the limiting distribution of M, where M = T/n, and what are the mean and variance of this limiting distribution? (4%).

(3). Assuming σ is known, please construct an approximated two-sided 95% confidence interval on μ based on M (5%)

(4). Please explain how you reach the above conclusions (2%)

5. An animal ecologist suggests three methods to prevent squirrels from damaging *Cryptomeria* plantations. To determine the effectiveness of the three methods, a small experiment was conducted with 9 replications for each treatment (method). The **Treatment Total** of each method is listed below. MSE of the experiment was 30. Please construct a complete ANOVA table based on the above information. Please state null and alternative hypotheses and your conclusions. Assuming the usual ANOVA assumptions apply. (20%)

	Method		
	1	2	3
Treatment Total	261	306	216

Table 1. Selected Critical Values of Standard Normal Distribution

z	0.0	1.645	1.96	2.325
$P(Z \geq z)$	0.5	0.10	0.025	0.01

Table 2. Selected Critical Values of F distribution for $\alpha = 0.05$

Numerator Degrees of Freedom

Denominator Degrees of Freedom

	1	2	3	4	5	6	7	8
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32
27	4.21	3.35	2.96	3.73	2.57	2.46	2.37	2.31
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28

試題隨卷繳回