

1. In the game of Parcheesi each player rolls a pair of dice on each turn. In order to begin the game, the player must roll a five on at least one dice, or a total of five on both dice. Find the probability that the player does not get to begin the roll on either their first or second rolls. (10%)
2. A machine pumps cleaner into a process at the rate of 7 gallons per minute. Upon inspection, it is learned that the machine pumps cleaner at a rate described by the uniform distribution over the interval 6.5 to 7.5 gallons per minute. Find the probability that the machine pumps less than 7 gallons during a randomly selected minute. (5%)
3. It is desired to estimate the proportion of college students who feel a sudden relief once their statistics class is over. How many students must be sampled in order to estimate the true proportion to within 2% at the 90% confidence level? (5%)
4. An economist wishes to study the monthly trend in the Dow Jones Industrial Average (DJIA). Data collected over the past 40 months were used to fit the model, where y = monthly close of the DJIA and t = month (1, 2, ..., 40). The regression results appear below:
 $\hat{y} = 88 + 0.25t$ $R^2 = 0.37$ $MSE = 144$ $F = 4.25$ $\text{Durbin-Watson } d = 0.96$
 Conduct a test to determine if the residuals are positively correlated (you need to state hypotheses, level of significance, decision rules, ..., etc). (10%)
5. A manufacturer of automobile transmissions uses three different processes. The manager ordered a study of the production costs to see if there is a difference among the three processes. A summary of the finding is show below.

	Process 1	Process 2	Process 3	Total
Process total (\$ 100's)	137	108	107	352
Sample size	10	10	10	30
Sum of squares	1,893	1,188	1,175	4,256

- (1) Which statistic do you use to perform the test? Explain in detail. (5%)
- (2) What is the variance explained by the different processes? (5%)
- (3) What is the variance unexplained by the different processes? (5%)
- (4) What is your conclusion for the test (you only need to point out the distribution of the test statistic, the decision rule, and degrees of freedom if it is necessary)? (5%)

6. In customer relationship management, marketing managers always keep eyes on their customer values. Two key variables are used to measure the customer values: the purchase amount per order and the interpurchase time between two orders. Let the random variables of Y be the purchase amount and, T , the interpurchase time. After taking the simple logarithmic transformation, i.e., $Y^* = \log(Y)$ and $T^* = \log(T)$, researchers suggest that the distributions of Y^* and T^* are normally distributed with mean, μ_Y and μ_T , and variance, σ_Y^2 and σ_T^2 , respectively. The correlation between Y^* and T^* is ρ . Then,
- (1) State the *name*, *format*, and the *pdf* of the joint distribution of Y^* and T^* . (10%) (Please see the note below)
 - (2) State the *name* and *format* of the joint distribution of Y and T . (5%)
 - (3) Why do we want to take the simple logarithmic transformation on Y and T ? (5%)
 - (4) State the *format* and derive the *pdf* of the conditional distribution of Y^* give $T^* = t^*$. (10%)
 - (5) How do you interpret the meaning of this conditional distribution ($Y|T=t$) to a marketing manager? (5%)
 - (6) Let $Z = \frac{Y}{T}$, State the *format* and derive the *pdf* of the distribution of Z . (10%)
 - (7) As a marketing manager, how do you use Z to help you analyze the customer values and develop relationship marketing strategies? (5%)

Note: An example of the terminologies of *name*, *format*, and the *pdf* could be:

Name: Normal distribution

Format: $X \sim \mathcal{N}(\mu, \sigma^2)$

$$\text{pdf: } f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$