

※ 注意：請於答案卷上依序作答，並標明大題及其題號。

A. (8 points for each of the following 10 blanks.)

◆ The radius of convergence of the series $\sum_{n=1}^{\infty} [\operatorname{csch}(n)] x^n =$ (1).

◆ If $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s$, and $z = 2r$, then $\frac{\partial w}{\partial r}$ can be expressed in terms of r and s as (2).

◆ The tangent plane at the point $(0, 1, 2)$ on the surface $\cos \pi x - x^2 y + e^{xz} + yz = 4$ is (3).

◆ The maximum value of the function $f(x, y, z) = x + 2y + 3z$ subject to the constraint $x^2 + y^2 + z^2 = 25$ is (4).

◆ By using Taylor's formula, a quadratic approximation of $f(x, y) = \cos x \cos y$ at the origin can be found to be (5).

◆ $\int_0^{\pi/2} \cos^3 x \sin 2x \, dx =$ (6).

◆ It is known that $f(x) + \int_0^x e^{2t} f'(t) \, dt = x$. Then $\lim_{x \rightarrow \infty} e^{2x} f'(x) =$ (7) and $\lim_{x \rightarrow \infty} f(x) =$ (8).

◆ $\int_0^1 \int_{y^2}^1 e^{\sqrt{x}} \, dx \, dy =$ (9).

◆ The value of a for which $\int_1^{\infty} (\frac{ax^2}{x^3+1} - \frac{1}{2x}) \, dx$ converges is (10).

B. (10 points)

◆ Use the definition of right-hand limit to show that $\lim_{x \rightarrow 1^+} (5x - 3) = 2$.

C. (10 points)

◆ Calculate the area enclosed by $y^2 = 2x$, $x + y = 4$, and $x + y = 12$. Please show your calculation.