※ 注意：請於答案卷上標明題號，並依序作答。

1. (5%)求取下列數列 $x_n (n = 1,2,3,\cdots)$ 的最大項之值 $x_n = \frac{n^2}{2^n}$

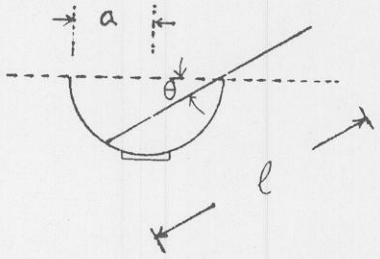
2. (5%)求取下列數列 $x_n (n = 1,2,3,\cdots)$ 的最小項之值 $x_n = n + \frac{100}{n}$

3. (10%)求下列式子的極限值 $\lim_{x\to 0}\left(\frac{1+x\cdot 2^x}{1+x\cdot 3^x}\right)^{\frac{1}{x^2}}$

4. (10%)求下列級數之和 $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{3\cdot 4\cdot 5}+\frac{1}{5\cdot 6\cdot 7}+\cdots$

5. (10%)在 $\frac{x}{a}+\frac{y}{b}=1$ 的條件下，求函數 $f(x,y)=x^2+y^2$ 的「極值」與「極值點座標」，並註明它是哪一類型的極值。

6. (10%)在一個半徑爲 a 的空心圓碗（像半球形）中，放置一根長度爲 l $(l > 2a)$ 的棒子，求棒子的平衡位置，也就是棒子與碗口平面的夾角。



7. (10%) A drug is carried into an organ of volume $V\ cm^3$ by a liquid that enters the organ at the rate of $a\ cm^3/sec$ and leave it at the rate of $b\ cm^3/sec$. The concentration of the drug in the liquid entering the organ is $c\ g/cm^3$. If the concentration of the drug in the organ at time t is increasing at the rate of $x'(t)=\frac{1}{V}(ac-bx_0)e^{-bt/V}$ $g/cm^3/sec$, and the concentration of the drug in the organ initially is $x_0\ g/cm^3$. What is the concentration of the drug at time t? _____

接背面

8. (10%) Find the maximum and minimum value of function $f(x,y)=2x-3y+1$ subject to the constraint $2x^2+3y^2-125=0$. The maximum value is _____; the minimum value is _____.

9. (10%) Based on data collected during an experiment, a biologist found that the number of fruit flies with a limited food supply could be approximately by the exponential model

$$N(t)=\frac{1000}{1+24e^{-0.02t}}$$

where t denotes the number of days since the beginning of the experiment. What is the average number of fruit flies in the colony in first 10 days of the experiment? _____; What is the average number of fruit flies in the colony in first 20 days of the experiment? _____.

10. (10%) The population density of a certain city is described by the function

$$f(x,y)=10{,}000e^{-0.2|x|-0.1|y|}$$

where the origin *(0,0)* gives the location of city hall. What is the population inside the rectangular area described by

$$R=\{(x,y)\,|\,-10\le x\le 10;\,-5\le y\le 5\}$$

if x and y are in miles? _____

11. (10%) Suppose x units of labor and y units of capital are required to produce

$$f(x,y)=100x^{3/4}y^{1/4}$$

units of a certain product. If each unit of labor costs \$200 and each unit of capital cost \$300 and a total of \$60,000 is available for production, how many units of labor and how many units of capital should be used in order to maximize production? Labor = _____ units; Capital = _____ units.

試題隨卷繳回