1. (40 分) Let x_n ($n=1,\ldots,N$) be N real numbers that are known. Let $f(\mu,\sigma^2)$ be the product of the n functions $\exp\{-\frac{(x_n-\mu)^2}{2\sigma^2}\}$ of μ and σ^2, with n running from 1 to N.

(a) Please find the maxima of $f(\mu,\sigma^2)$ in the region $R_C=\{(\mu,\sigma^2): \mu\le 0 \text{ and } \sigma^2\ge 1\}$. Then, please find the global maximum in R_C of the above problem.

(b) Please do (1) in the region $R_O=\{(\mu,\sigma^2): \mu< 0 \text{ and } \sigma^2> 1\}$. Pease do (2) in the region R_O.

(c) Please do (1) in the region $R_U=\{(\mu,\sigma^2): \mu\le 0 \text{ or } \sigma^2> 1\}$. Please do (2) in the region R_U.

2. (10 分) Please show that $f(x)=(1+x)^{1/x}$ is decreasing on $(0,\infty)$.

3. (10 分) Please show that $e=\lim_{h\to 0}(1+h)^{1/h}$.

4. (10 分) For any given x, $0\le x\le 1$, define the function $f(y\mid x)=y\cdot x^{y-1}$, $0<y<\infty$. Please find y^*, such that $f(y\mid x)$ is maximized at y^*.

5. (10 分) Let matrix $P=\begin{pmatrix} p^2 & 2pq & q^2 \\ p^2 & 2pq & q^2 \\ p^2 & 2pq & q^2 \end{pmatrix}$, where $p+q=1$. Please show that P is idempotent. That is, to show that $P^2=P$.

6. (10 分) Let matrix $A=\begin{pmatrix} 1 & 1/2 \\ 0 & 1/2 \end{pmatrix}$, please find A^n.

7. (10 分) Let matrix $B=\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$, please find its determinant, inverse, and eigenvalues.