1. Pick two integers randomly from $5 \leqq x \leqq 9$, $11 \leqq y \leqq 19$
   (a) What is the sample space of this experiment? (5%)
   (b) What is the probability that $x$ and $y$ are relative prime? (5%)

2. A fair coin is tossed $n$ times. Show that the events "at least two heads" and "one or two tails" are independent if $n = 3$, but are dependent if $n = 4$. (15%)

3. In a memoryless binary asymmetric channel, the transmitted bits will be reversed randomly and let the probability that the transmitted bit is flipped from '0' to '1' be $p$ and the probability that the transmitted bit is flipped from '1' to '0' be $q$, where $0 < p, q < 1$. If 40% of the transmitted bits are '0' and 60% are '1', what is the probability that a received '0' is actually a '1'? (15%)

4. Let $X$ be a random variable with expected value $\mu$ and variance $\sigma^2$, then for any $t > 0$
   (a) State the Chebyshev's inequality (5%)
   (b) Prove this inequality (10%)

5. If $Y$ and $Z$ are two Gaussian random variables with unit variance and mean $a$ and $b$, respectively and

$$X = \begin{array}{ll} Y & \text{with probability } p \\ Z & \text{with probability } 1\text{-}p, \end{array}$$

where $0 < p < 1$. Find the expectation and variance of $X$. (15%)

6. For a random variable $X$ with moment-generating function $M_X(t) = (1/81)(e^t + 2)^4$, find $P(X > 2)$. (15%)

7. Let $X$ be a continuous random variable with the probability density function
$$f(x) = ce^{-|x|} \qquad \text{if } |x| \geqq 1$$
$$= 0 \qquad \text{otherwise}$$
   (a) Find $c$. (5%)
   (b) Find the probability density function of $Y$ if $Y = X^2$. (10%)

試題隨卷繳回