1. In this problem, we consider that a signal is given by $x(t) = 6\cos(10\pi t)$. Let $x(t)$ be sampled at f_s Hz to produce a sampled signal $x_s(t)$.
   (a) Find the expression for the spectrum $X_s(f)$ of $x_s(t)$ and plot $X_s(f)$ in detail if $f_s = 7$. (7%)
   (b) Repeat (a) if $f_s = 14$. (4%)
   (c) Explain how to reconstruct the $x(t)$ from $x_s(t)$ for (a) and (b). You should justify your answer. (4%)

2. In this problem, we want to analyze the spectrum of a transient signal $x(t)$. Assume that we measure $x(t)$ and record $x(t)$ with a 2×10^{-3} second duration. In order to avoid significant aliasing, a sampling rate of 5 kHz is required to take samples from $x(t)$. Let the desired spectral resolution be 100 Hz for analyzing the signal spectrum.
   (a) Is zero padding necessary to achieve the desired resolution? You should justify your answer. (7%)
   (b) Find the number of fast Fourier transform (FFT) points required for computing the discrete Fourier transform (DFT). (8%)

3.(a) Draw the block diagrams of the transmitter and receiver respectively for a differential BPSK system, where carrier recovery is not used in the receiver. (6%)
   (b) Describe the advantage and disadvantage of using differential MPSK. (4%)

4. Consider a (255,233) Reed-Solomon code over GF(2^8)
   (a) What is its codeword length in bits? (2%)
   (b) What is its code rate? (2%)
   (c) What kind of errors can be corrected by this code? (3%)

5.(a) What is the unit of channel capacity for a binary symmetric channel? (2%)
   (b) What is the unit of channel capacity for an additive white Gaussian noise channel? (2%)
   (c) Consider a binary symmetric channel with transition probability $p = 1/2$. What is its channel capacity ? Is it possible to design an error-correcting code to reduce its decoding error probability close to zero? (4%)

6.(a) Find the impulse response of a duobinary signaling scheme which has a frequency response of a half-cycle cosine shape. (4%)
   (b) Describe the necessity of precoding for such a duobinary signaling scheme. (3%)
   (c) Describe the advantage and disadvantage of using duobinary signaling scheme (3%)

7. Let $N(t)$ be a while Gaussian noise process with power spectral density equal to $N_0/2$.
   (a) Please show that if the input signal of a matched filter is $N(t)$, then the output samples of the matched filter is always a sequence of independent Gaussian random variables. (10%)
   (b) Suppose $X(t) = N(t) + c\,N(t-d)$, where d is a constant. Please derive the autocorrelation function and the power spectral density of $X(t)$, respectively. (12%)

8. Consider a typical coherent Binary Phase Shift Keying system, with signals

$$S_1(t) = \sqrt{(2E_b/T)}\cos\omega_0 t \qquad \text{for } 0 \le t \le T.$$

$$S_0(t) = -\sqrt{(2E_b/T)}\cos\omega_0 t \qquad \text{for } 0 \le t \le T,$$

Where Eb is the energy per bit, T is the symbol duration, and ω_0 is the carrier frequency.

If the noise is Gaussian but is slowly varying, and the noise spectral density is uniformly distributed over $(0, N_0/2)$. Please derive its symbol error probability. (13%)