1. Let $\mathbb{F}_9$ be a finite field of nine elements. Can you find a $3\times 3$ matrix with entries in $\mathbb{F}_9$ but one of its eigenvalues is in an extension field of $\mathbb{F}_9$ rather than in $\mathbb{F}_9$ itself? (25/100)

2. $\exp A = \text{identity} + A + \frac{1}{2}A\cdot A + \frac{1}{3!}A\cdot A\cdot A + \cdots$

$$A = \begin{pmatrix} 9 & -4 & -6 \\ -9 & 9 & 9 \\ 12 & -8 & -9 \end{pmatrix} \qquad \exp A = \begin{pmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{pmatrix} \qquad (25/100)$$

3. $\vec{v}_1 = (1,2,3,4)$, $\vec{v}_2 = (4,3,2,1)$, $\vec{v}_3 = (1,3,2,4)$ If you can find an orthogonal matrix $A$, $A\cdot A^T = I$ so that $\vec{v}_1\cdot A = (u_{11}, u_{12}, u_{13}, 0)$, $\vec{v}_2\cdot A = (u_{21}, u_{22}, u_{23}, 0)$ $\vec{v}_3\cdot A = (u_{31}, u_{32}, u_{33}, 0)$, then $\det(u_{ij}) = ?$ (25/100)

4. Can you find a $2\times 2$ unitary matrix $U$, $\overline{U}\,U^T = I$ and $\det U \neq \pm 1$? If not, explain why not. (25/100)

試題隨卷繳回