

1. The beam of length  $L$  shown in Figure 1(a) has a uniform cross-sectional property, i.e.,  $EI = \text{constant}$ . It is built in the wall on the left and simply supported on the right. The deflection  $w(x)$  and transverse load per unit length  $q(x)$  are shown in the positive direction. The sign conventions of a positive bending moment  $M(x)$  and a transverse shear force  $V(x)$  are given in Figure 1(b). (Total 23%)

- (a) Given  $\frac{d^2 w}{dx^2} = \frac{M}{EI}$ , show that  $\frac{d^4 w}{dx^4} = \frac{q}{EI}$  using the condition of equilibrium. (7%)
- (b) Write down the boundary conditions at  $x = 0$  and  $x = L$  of the beam. (4%)
- (c) If  $q(x) = q_0$ ,  $q_0$  is a constant, determine the deflection  $w(x)$  of the beam. (8%)
- (d) Draw the shear force and bending moment diagrams using the deflection  $w(x)$  obtained in (c). (4%)

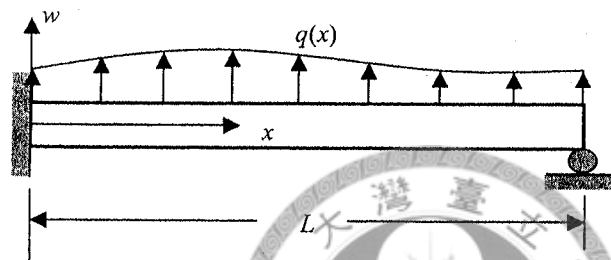


Figure 1 (a)

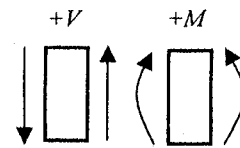


Figure 1(b)

2. When a beam with a uniform cross-sectional property ( $EI = \text{constant}$ ) is axially loaded by a compressive force  $P$ , the equation of equilibrium can be written as  $\frac{d^4 w}{dx^4} + \frac{P}{EI} \frac{d^2 w}{dx^2} = \frac{q}{EI}$ , where  $q(x)$  is the distributed load on the transverse direction. When  $q(x) = 0$ , the compressive force  $P$  that makes  $w(x) \neq 0$  is called the critical load. Determine the critical load for the beam shown in Figure 2. (12%)

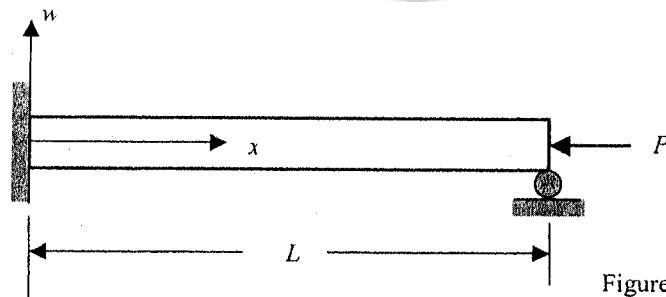


Figure 2

3. A stiff beam of length  $2a$  is hinged at one end and supported by two springs of spring constant  $k$  as shown in Figure 3. Where should a force  $P$  be applied so that the stiffness of the system (defined as  $P$  divided by the deflection under  $P$ ) is  $20k/9$ ? Please express the answer  $c$  in terms of  $a$ . (10%)

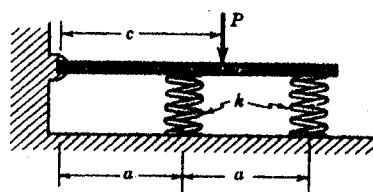


Figure 3

接背面

4. The bar shown in Figure 4 consists of material with shear modulus  $G = 28 \text{ GPa}$  and has a solid circular cross section. Part  $A$  is 40 mm in diameter and Part  $B$  is 20 mm in diameter. (Total 25%)
- Draw the torque diagram of the bar. (5%)
  - Determine the magnitudes of the maximum shear stresses in Parts  $A$  and  $B$ . (10%)
  - Determine the location where maximum (absolute) angle of twist occurs and find its value in degrees. (10%)

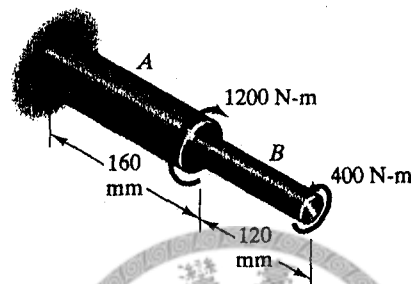


Figure 4

5. The pressure in a cylindrical tank as shown in Figure 5 is monitored by a  $45^\circ$  strain rosette mounted on its surface. Unfortunately, during service, gages  $a$  and  $c$  have become inoperative. Only gage  $b$  gives a strain reading of  $100 \times 10^{-6}$ . If  $d = 1 \text{ m}$ ,  $t = 10 \text{ mm}$ ,  $E = 207 \text{ GPa}$ , and  $\nu = 0.3$ , find the pressure in the tank. (15%)

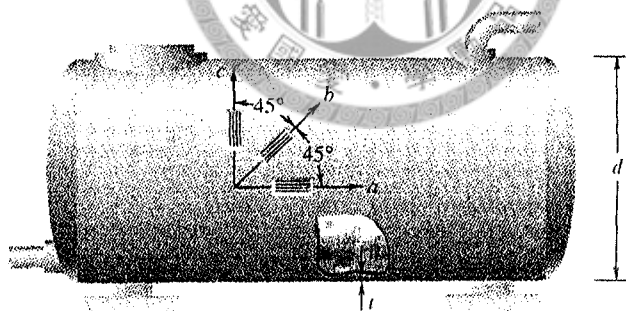


Figure 5

6. A section of pressurized steel pipe experiences a twisting moment  $T$  as shown in Figure 6. If  $T = 100 \text{ kip-in}$ , determine the largest  $p$  that the pipe can sustain before point  $A$  yields (use any criterion you like). The outside diameter of the pipe is 12 in, and the wall thickness is 0.125 in. (15%)

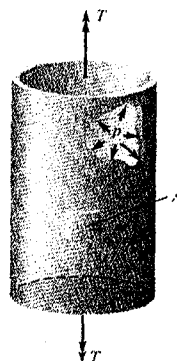


Figure 6