1. Find the general solutions of the following differential equations of $y(x)$:

(i) $y' - 3y = x$

(ii) $\dfrac{d^4 y}{dx^4} - y = 0$

(iii) $x^3 y''' + x^2 y'' - 2xy' + 2y = 0$

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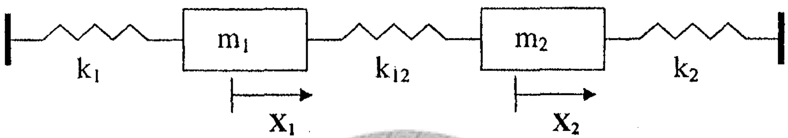
2. Consider Mass-Spring system



If $m_1 = m_2 = k_1 = k_{12} = k_2 = 1$ and the initial conditions are

$$x_1(0) = x_2(0) = 1$$
$$\dot{x}_1(0) = \dot{x}_2(0) = 0$$

Find $x_1(t)$ and $x_2(t)$

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3. Given the Sturm-Liouville problem of $y(x)$:

$$y'' + \lambda^2 y = 0 \qquad 0 \le x \le L$$
$$y(0) = 0$$
$$y'(L) + Ay(L) = 0$$

Where L, A are constants. Find the eigenvalues and the normalized eigenfunction of the problem.

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4. Solve the integral equation of convolution type

$$y(t) = t^2 + \int_0^t y(\tau)\sin(t-\tau)\,d\tau$$

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5. Find the steady state solution of the following wave equation of $u(x,t)$: $\quad u_{tt} - c^2 u_{xx} = 0 \qquad 0 \le x \le L;\ 0 \le t$

the boundary conditions are:

$$u(0,t) = A\sin(\omega t)$$
$$u(L,t) = 0$$

where c, A, ω are constants. What restrictions must be placed on ω ?

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