

※ 注意：請於答案卷上依序作答，並應註明作答之題號。

• 請勿將第 1 至 4 題(填充題)之答案寫於本試題紙上！

1. (7%) If  $X$  is a continuous random variable uniformly distributed over  $(0, 1)$ , then the variance of  $X^4$  is (1).
2. (8%) If  $X$  has the normal distribution with mean  $\mu$  and variance  $\sigma^2$ , then the expected value of  $X^4$  is (2).
3. (8%) Let  $X$  and  $Y$  be independent random variables each having the normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Also let  $U=X-3Y$  and  $V=3X+Y$ . Then the joint probability density function of  $U$  and  $V$ ,  $f_{U,V}(u,v)$ , is (3).
4. (7%) According to Chebyshev's inequality, the probability that in  $n$  throws of a fair die the number of fives lies between  $n/6-(2n)^{1/4}$  and  $n/6+(2n)^{1/4}$  is greater than or equal to (4).

• 請將第 5 至 8 題(問答計算題)之答案及計算過程寫於答案卷上！

5. (15%) Independent trials, each resulting in a success with probability  $1/3$ , are successively performed. Let  $N$  be number of trials performed to have the first success. Find the variance of  $N$ .
6. (15%) Let the pair of random variables  $X$  and  $Y$  have the standard bivariate normal distribution with parameter  $\rho$ . In other words, the joint probability density function of  $X$  and  $Y$  is

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)\right).$$

Find the conditional density function of  $Y$  given that  $X=x$ .

接背面

7. (16%) Suppose that from the response of 210 consumers, 150 of them engaged in negative word of mouth behavior. On average, these people spoke to 4.9 people about their dissatisfying experience (with a standard deviation equal to 6.1) with some retailers.
- Compute a 90 percent confidence interval for the proportion of all consumers who engage in negative word-of-mouth behavior. Would it be reasonable to claim that more than 75 percent of all consumers engage in such behavior?
  - Compute a 90 percent confidence interval for the mean number of people who are told about a dissatisfying experience by those who engage in negative word-of-mouth behavior. Would it be reasonable to say that dissatisfied consumers spread their bad experience at least to four people?
8. (24%) For bottling companies, both overfilling and under-filling bottles are undesirable as under-filling leads to customer dissatisfaction and overfilling adds up extra cost. The mission of quality management is to see whether the mean bottle fill,  $\mu$ , is close to the target of 12 ounces.
- Set up the null and alternative hypotheses for this hypothesis test to help the bottling company so that the filler will be readjusted if the null hypothesis is rejected.
  - In this situation, interpret what aspect constitutes making a Type I error and making a Type II error respectively.
  - Given a level of significance of  $\alpha = .05$ , determine the rejection points needed to test the null and alternative hypotheses.
  - Suppose that a random sample of 54 bottle fills is taken from a test run of the filler. For each of the following sample results, determine whether the filler's initial setup should be readjusted.
    - $\bar{x} = 12.05$  and  $\delta = .1$
    - $\bar{x} = 11.96$  and  $\delta = .1$