國立台灣大學九十三學年度碩士班招生考試試題

科目：微積分(不含線性代數)　　　　題號：405

共 一 頁之第 全 頁

注意：請於答案卷上依序作答，並標明大題及其題號

A.（8 points for each of the following 9 blanks.）

- $\lim_{x\to 0}\frac{\sin(2x)-2x}{x^3} =$ __(1)__ .
- When $[f(x)]^2 = 36 + \int_0^x \{[f(t)]^2 + [f'(t)]^2\}dt$, it can be shown that $f(x) = af'(x)$. Then $a =$ __(2)__ .
- The highest and lowest points on the curve $x^2 + xy + y^2 = 12$ are __(3)__.
- $\int_0^3 \frac{dx}{x^2-x-2} =$ __(4)__.
- The radius of convergence of the series $\sum_{i=1}^{\infty}\frac{(nx)^n}{n!}$ is __(5)__
- $\int_0^1\int_y^1 e^{\max\{x^2,y^2\}}dxdy =$ __(6)__ where $\max\{x^2,y^2\}$ means the larger of the numbers x^2 and y^2.
- It is known that $u = \sin(x-2t) + \ln(x+2t)$ is a solution of the equation $\partial^2 u/\partial t^2 = c\partial^2 u/\partial x^2$. Then $c=$ __(7)__.
- The total production P of a certain product depends on the amount L of labor used and the amount K of capital investment. When $P = cL^{\alpha}K^{1-\alpha}$, the maximum production occurs at $mL=\alpha p$ and $nK=$ __(8)__ p if $mL+nK=p$.
- Define $I(r) = \iint_D \frac{1}{x^2+y^2}\,dA$ where D is the region bounded by the circles with center the origin and radius R and 1, $0<R<1$. $\lim_{R\to 0} I(R) =$ __(9)__.

B.（14 points）

- Find the maximum and minimum of $f(x,y) = 2x^3 + y^4$ over the region defined by $D = \{(x,y): x^2+y^2 \le 1\}$.

C.（14 points）

- Consider the function

$$f(x,y,z) = \begin{cases} \frac{(x+y+2z)^{\alpha}}{x^2+y^2+z^2} & \text{if } (x,y,z) \ne (0,0,0) \\ 0 & \text{if } (x,y,z) = (0,0,0) \end{cases}.$$

Prove that $f(x,y,z)$ is continuous at (0,0,0) when $\alpha > 2$ and explain why $f(x,y,z)$ is not continuous at (0,0,0) when $\alpha = 2$.

試題隨卷繳回