※ 注意：請於答案卷上依序作答，並應註明作答之題號。

1. (25 points) Consider a binary search tree. Each data node has a key assumed a positive integer.

   (a). Give C or C++ data representation of the binary search tree using dynamic memory allocation and linked list representation.

   (b). Write a pseudo algorithm that outputs all data nodes in the binary search tree according to their key values in decreasing order. The time complexity of your algorithm must be constrained by O(N), where N is the number of nodes of the tree.

2. (25 points) Consider a project management problem. We can represent such project management problem by a graph. A number of tasks to be performed on a project are edges. Events which signal the completion of certain tasks are vertices. An event occurs only when all tasks entering it have been completed. The figure below is an example of 11 tasks, $a_0, ..., a_{10}$, and eight events $v_0, ..., v_7$. Let v_0 and v_7 be "start project" and "finish project," respectively. The number associated with each task is the time required to perform the task. Thus, task a_0 requires 5 days. Tasks a_0 and a_1 can be performed concurrently after the start of the project. However, tasks a_2, a_3 and a_4 cannot be started until event v_1 occurs after task a_0 has been completed.

   (a). Give C or C++ data representation of this task-event graph using adjacency list.

   (b). Write a pseudo algorithm to find the minimum amount of time required to complete a project.
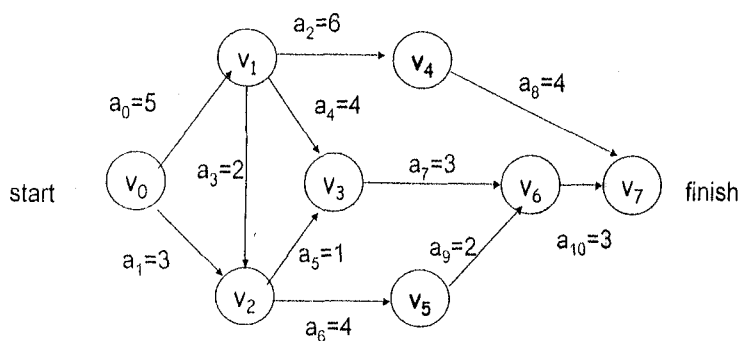
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3. (20 points) There is a small town of N people where each person can be uniquely identified by an integer from 1 through N. In this town, if a person X is a friend of a friend of another person Y, then X is also considered a friend of Y. Mathematically, the "friend" relation is reflexive, symmetric, and transitive, and hence is an equivalence relation. The population of the town is therefore partitioned by the relation into groups where people in the same group are friends of each other.

You are now given the number N of people and a list of pairs of the form (A, B) where $1 \leq A, B \leq N$ indicating that the two persons A and B are friends; the list is not necessarily exhaustive as the friendship of two persons may be inferred from other friendships. Your task is to design an algorithm that determines the number of people in the largest group of friends. Please present your algorithm in an adequate pseudo code and make assumptions wherever you feel necessary. Explain why your algorithm is correct and analyze its time complexity. The more efficient your algorithm is, the more points you will be credited for this problem.

4. (20 points) A string $A = a_0 a_1 \cdots a_{n-1}$ is a cyclic shift of another string $B = b_0 b_1 \cdots b_{n-1}$ if there exists an index k, $0 \leq k \leq n-1$, such that $a_i = b_{(k+i) \bmod n}$ for all i, $0 \leq i \leq n-1$. For example, "defgabc" is a cyclic shift of "abcdefg". Suppose that you already have an algorithm called **Substring** that can determine whether a string is a substring of another. Design an algorithm using **Substring** that, given two strings A and B, determines whether A is a cyclic shift of B. Please present your algorithm in an adequate pseudo code and make assumptions wherever you feel necessary. Explain why your algorithm is correct and analyze its time complexity. The more efficient your algorithm is, the more points you will be credited for this problem.

5. (10 points) Among the most well-known NP-complete problems are the SAT, Clique, Hamiltonian Cycle, Travelling Salesman, Partition, and Knapsack problems. Please describe as precisely as possible any two of them. Explain how you can prove the NP-completeness of one of the two problems, given that the other has been proven NP-complete.

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