1. (11%) (a) (7%) Use the Laplace transform to solve the given initial value problem.

$$y''+16y=f(t), y(0)=0, y'(0)=1, \text{where } f(t)=\begin{cases}\cos 4t, & 0\le t<\pi \\ 0, & t\ge\pi\end{cases}$$

(b) (4%) Solve the given integrodifferential equation

$$y'(t)=\cos t+\int_0^t y(\tau)\cos(t-\tau)d\tau, y(0)=1$$

2. (7%) Solve the wave equation subject to the given conditions.

$$a^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2},\quad 0<x<1,\quad t>0$$

$$u(0,t)=0,\quad u(1,t)=0,\quad u(x,0)=0.01\sin(3\pi x),\quad \left.\frac{\partial u}{\partial t}\right|_{t=0}=0$$

3. (7%) Use the power series method to solve the given initial value problem.

$$(x+1)y''-(2-x)y'+y=0,\quad y(0)=2,\quad y'(0)=-1$$

4. (15%) Solve the following differential equation

$$x\frac{dy}{dx}=y(x^2y-1)$$ (hint: a Bernoulli differential equation)

5. (10%) A one-parameter family of solutions for some differential equation is

$$y=\frac{3-2ce^{2x}}{1-ce^{2x}}$$

Please find the differential equation and its singular solution.

6. (25%) Let $A=\begin{bmatrix}1 & -2 & -1 & -4\\ 2 & -4 & 3 & 7\\ -2 & 4 & 1 & 5\\ -3 & 6 & -8 & -21\end{bmatrix}$ and $\mathbf{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$.

(a) (10%) Find a spanning set for the solutions to $A\mathbf{x}=\mathbf{0}$.

(b) (5%) What is the determinant of A?

(c) (10%) Let a linear transformation $T:R^4\to R^4$ be defined as $T(\mathbf{x})=A\mathbf{x}$. Determine the matrix representation of T with respect to the following basis $B=\left\{\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}0\\1\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\begin{bmatrix}2\\1\\0\\0\end{bmatrix}\right\}$.

7. (25%) Consider the following matrix:

$$\mathbf{A}=\begin{bmatrix}0 & 1 & 2\\ 1 & 0 & 1\end{bmatrix},$$

(a) (5%) Please find the singular values of $\mathbf{A}$;

(b) (10%) Please find the orthonormal bases of right and left singular vectors of the matrix $\mathbf{A}$;

(c) (10%) Please find the pseudoinverse of the matrix $\mathbf{A}$.

試題隨卷繳回