1. (15%) Find

   (a)

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$$

   (b)

$$\lim_{x\to 0}\frac{\ln\sin 2x}{\ln\sin x}$$

   (c)

$$\lim_{x\to 0} x^x$$

2. (10%) Find

$$\int \frac{\sqrt{x}}{x-1}dx$$

3. (20%) A mass m is projected vertically upward from the ground with initial velocity v_0. Find the maximum height reached, assuming that the resistance of the air is proportional to the velocity, i.e., Kv where K is a constant.

4. (15%) Solve

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}=\sin 3x$$

   where y is a function of x.

5. (20%) Let

$$A=\begin{bmatrix}0 & -2\\ 1 & 3\end{bmatrix}$$

   Compute A^n for any positive integer n. (Hint: Find a diagonal martix P and an invertible matrix Q such that $A=QPQ^{-1}$.)

6. (20%) The time taken by a garage to repair a car is a continuous random variable with the probability density function

$$f_X(x)=\begin{cases}\frac{3}{4}x(2-x) & 0\le x\le 2\\ 0 & \text{elsewhere}\end{cases}.$$

   If, on leaving his car, the driver goes to a party lasting for a time Y where Y is a continuous random variable independent of X, with probability density function

$$f_Y(y)=\begin{cases}\frac{1}{2}y & 0\le y\le 2\\ 0 & \text{elsewhere}\end{cases}$$

   Determine the probability that the car will not be ready on his return.

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