

1. Find a binary tree whose preorder and inorder traversal create the same result (10 points)
2. Which of the following sequences are max-heaps? (10 points)
  - a. 42 35 37 20 14 18 7 10
  - b. 42 35 18 20 14 30 10
  - c. 20 20 20 20 20 20
3. The incidence matrix of a directed graph  $G = (V, E)$  is a  $|V| \times |E|$  matrix  $B = (b_{ij})$  such that
 
$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i, \\ 1 & \text{if edge } j \text{ enters vertex } i, \\ 0 & \text{otherwise.} \end{cases}$$

Let matrix  $C = BB^T$ . Describe what the entries of the matrix  $C$  represent. (10 points)
4. If the values of  $A, B, C$ , and  $D$  are 2, 3, 4 and 5, respectively, manually calculate the value of the following prefix expressions. (10 points)
  - a.  $+-\times ABCD$
  - b.  $-\times A + BCD$
5. Show the tree representation of the following parenthetical notation:  $a(b(c d) e f(g h))$  (10 points)
6.  $M$  is an  $n \times n$  integer matrix in which the entries of each row are in increasing order (reading left to right) and the entries in each column are in increasing order (reading top to bottom). Give an efficient algorithm to find the position of an integer  $x$  in  $M$ , or determine that  $x$  is not there. Tell how many comparisons of  $x$  with matrix entries your algorithm does in the worst case. (25 points)
7. Give an  $O(|V|(|E| + |V|))$ -time algorithm for computing the transitive closure of a directed graph  $G = (V, E)$ . (25 points)