國立台灣大學九十三學年度碩士班招生考試試題

科目：工程數學(D)

題號：455

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1. (20%) Label the following statements as being true or false. (No explanation is needed. Each correct answer gets 2% and each wrong answer gets 0%):

   (a) If an $n \times n$ matrix is not invertible, then it has an eigenvector in $\mathcal{R}^n$.

   (b) We can always define infinitely many inner products for an inner product space.

   (c) Let $A \in \mathcal{R}^{m\times n}$ and R be its reduced row echelon form. Then the span of column of R is equal to the span of columns of A.

   (d) If two $n \times n$ matrices have the same characteristic polynomial, then they are similar.

   (e) If $\mathcal{S}$ is a linearly independent subset such that every vector in V can be written as a linear combination of the vectors in $\mathcal{S}$. Then $\mathcal{S}$ is a basis for V.

   (f) Let A be an $n \times n$ matrix and $\| A\mathrm{v} \| = \| \mathrm{v} \|$ for every v in $\mathcal{R}^n$. Then A is orthogonal.

   (g) *Every* matrix in $\mathcal{M}_{5\times 5}(\mathcal{R})$ has an eigenvector in $\mathcal{R}^5$.

   (h) Given any A in $\mathcal{M}_{m\times n}(\mathcal{R})$, AA^T is always diagonalizable.

   (i) Let A and B be $n \times n$ matrices. Suppose that B is not invertible. Then rank $AB <$ rank A..

   (j) Let $\{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3, \mathrm{v}_4\}$ be a linearly independent subset of $\mathcal{R}^5$ and let $T : \mathcal{R}^5 \to \mathcal{R}^4$ be linear. Then $\{T(\mathrm{v}_1), T(\mathrm{v}_2), T(\mathrm{v}_3), T(\mathrm{v}_4)\}$ cannot be a linearly independent subset of $\mathcal{R}^4$.

2. Let $V = \mathcal{M}_{2\times 2}(\mathcal{R})$ be an inner product space with $< A, B >= \mathrm{trace}(AB^T)$.

   (a) (6%) Let W be the subspace of V containing all symmetric 2×2 matrices. Find an orthonormal basis for W.

   (b) (4%) Find a basis for $W^{\perp}$.

   (c) (5%) Find the matrix in W that is closest to $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$.

3. Find the reduced row echelon form of A, B and C defined below.

   (a) (5%) A is a 5×4 matrix whose columns are linearly independent.

   (b) (5%) $B = \mathrm{u}\mathrm{v}^T$, where u is a nonzero vectors of $\mathcal{R}^n$ and $\mathrm{v}^T = [1 \ 2 \ 3 \ \ldots \ n]$.

   (c) (5%) $C = R^T$ where R is the reduced row echelon form of a 4×5 matrix with rank = 3.

4. Let X and Y be two independent random variable and both are uniformly distributed over [-1, 1]. Please determine the following statement as being true or false. Derivation is required.

   (a) Let A= X+ Y and B = X- Y, then the probability density function of A and B are identical. (8%)

   (b) Let Z be a normal random variable with zero mean and variance 1/4. Then under Chebychev's inequality, one can argue that Pr{ $|Z| \geq 1 + |X|/2$ } $\leq 1/4$. (8%)

5. Let X and Y be two independent Poisson random variables with mean equal to λ_1 and λ_2 respectively. Please dervie the formula for E[X | X+ Y = z], where z is a non-negative integer. (10%)

6. Let X and Y be two independent geometric random variables, both mean equal to q. (i)Please find the covariance Cov(X+Y, X-Y) (ii) Please find the joint probability function of X+Y and X-Y. (12%)

7. Let X_1, X_2, ….X_N be a set of independent random variables, where each X_i is a normal random variable with mean equal to μ and variance equal to σ^2. Please derive the moment generating function of Y. , where Y= $X_1 + X_2$ ….+ X_N and N is a Poisson random variable with mean λ. (12%)

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