1.  Let T be a torus, $\pi_1(T)$ its fundamental group. $f: T \to X$ is a continuous map from T to X which is path-connected, locally path-connected and locally simply connected, and $\pi_1(X)$ is its fundamental group. Can you prove $\pi_1(T)$ is a subgroup of $\pi_1(X)$? If yes, can you prove it is a normal subgroup? $(25/100)$

2. $x = t^3$, $y = 3t$, $z = t^4$ is a space curve. Can you find an osculating plane parallel to the plane $x+y+z=1$? $(25/100)$

3. Let $x = x(r,\theta)$, $y = y(r,\theta)$, $z = z(r,\theta)$ $0<r<\infty$ $-\infty<\theta<\infty$ be a surface. $E = \left(\frac{\partial x}{\partial r}\right)^2 + \left(\frac{\partial y}{\partial r}\right)^2 + \left(\frac{\partial z}{\partial r}\right)^2 = 2$
$F = \frac{\partial x}{\partial r}\frac{\partial x}{\partial \theta} + \frac{\partial y}{\partial r}\frac{\partial y}{\partial \theta} + \frac{\partial z}{\partial r}\frac{\partial z}{\partial \theta} = 0$, $G = \left(\frac{\partial x}{\partial \theta}\right)^2 + \left(\frac{\partial y}{\partial \theta}\right)^2 + \left(\frac{\partial z}{\partial \theta}\right)^2 = r^2$
At the point $(r,\theta) = (1,0)$ find its mean curvature $H = ?$ If you claim it is impossible to find H, you need to give at least two surfaces with different H at $(r,\theta) = (1,0)$ both satisfy the above conditions. $(25/100)$

4. Let $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$ $-\infty<u,v<\infty$ be a surface. $E = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2 = 1+u^2+4u^2v^2-4u^2v$
$F = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v} = 2uv - u^3 - 4uv^2 + 2u^3v$
$G = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = 1+4v^2-4u^2v+u^4$ At the point $(u,v) = (0,0)$ find its Gauss curvature $K = ?$ If you claim it is impossible to find K, you need to give at least two surfaces with different K at $(u,v) = (0,0)$, both satisfy the above conditions $(25/100)$