請依照題號順序作答。所有數字必須化為最簡分數，未依規定者該題不予計分。

1.填充題.請將空格編號(A),(B),(C),...,(K)以及答案依順序填寫於答案卷上。(A)至(F)每題 7 分,(G)至(K)每題 6 分，合計 72 分。

(a) The equation of the normal line to the curve $x^2+y^2=(2x^2+2y^2-x)^2$ at $(0,1/2)$ is (A).

(b) Evaluate the following limits.

$$(i)\ \lim_{x\to\infty}\sqrt{x}e^{-x}\int_3^{\sqrt{x}}e^{t^2}\,dt=\underline{(\mathbf{B})},\qquad (ii)\ \lim_{x\to\infty}\left(\frac{x+1}{x+2}\right)^{3x+4}=\underline{(\mathbf{C})},$$

$$(iii)\ \lim_{n\to\infty}n^{-2}\sum_{k=1}^{n}ke^{-k/n}=\underline{(\mathbf{D})}.$$

(c) By reversing the order of integration, the iterated integral $\int_0^4\left(\int_{\sqrt{y}}^2 e^{x^3}\,dx\right)dy$ can be evaluated and is equal to (E).

(d) By applying the transformation $x=\frac{u}{\sqrt{3}}+v$, $y=\frac{u}{\sqrt{3}}-v$, the integral $\iint_D e^{x^2+xy+y^2}\,dA$, where $D=\{(x,y)\,|\,x^2+xy+y^2\le 3\}$, can be evaluated and is equal to (F).

(e) When the first **TWO** nonzero terms of the Taylor series of e^{-x^2} is used to approximate the integral $\int_0^{1/10}e^{-x^2}\,dx$, the approximation is (G), and the error is at most (H).

(f) The function $f(x,y)=\frac{1}{3}y^3+2x^2y+\frac{1}{2}y^2+4xy$ has a local maximum at the point(s) $(x,y)=$ (I), a local minimum at the point(s) $(x,y)=$ (J), and saddle point(s) at $(x,y)=$ (K).

計算題 2 題，每題 14 分。

2. First show that the improper integral $\int_0^\infty \frac{e^{-3x}}{\sqrt{x}}\,dx$ is convergent. Then evaluate this integral.

3. Use Lagrange multipliers to find the points on the curve $x^2-xy+y^2=2$ that are closest to and farthest from the origin.

試題隨卷繳回