

A. (8 points for each of the following 9 blanks.)

- $\lim_{x \rightarrow \infty} x \frac{\int_x^{\infty} e^{-t^2/6} dt}{e^{-x^2/6}} = \underline{(1)}$.
- $\lim_{n \rightarrow \infty} \left(1 - \frac{\cos(3/n)}{2n}\right)^n = \underline{(2)}$.
- When $dg(x)/dx = 1 + [g(x)]^2$ and $g(0) = 0$, then $g(x) = \underline{(3)}$.
- When $a = \underline{(4)}$, $\int_1^{\infty} \left(\frac{ax}{x^2+x+1} - \frac{1}{2x}\right) dx$ converge.
- Find $ab = \underline{(5)}$ such that $\int_a^b (24 - 2x - x^2)^{1/3} dx$ has its largest value.
- $\int_1^e \int_1^x \int_0^{2y} dy dz dx = \underline{(6)}$.
- It is known that $r = \sqrt{x^2 + y^2 + z^2}$. Then $r(\partial^2 r / \partial x^2 + \partial^2 r / \partial y^2 + \partial^2 r / \partial z^2) - ((\partial r / \partial x)^2 + (\partial r / \partial y)^2 + (\partial r / \partial z)^2) = \underline{(7)}$.
- The plane $z = Ax + By + C$ is said to be fitted to the points $(x_1, y_1, z_1), \dots, (x_n, y_n, z_n)$ when A, B and C minimize $\sum_{i=1}^n (Ax_i + By_i + C - z_i)^2$. Due to the nature of the problem, it is known that $A \leq 10$. When $\sum_{i=1}^n x_i = 0$, $\sum_{i=1}^n y_i = 0$, and $\sum_{i=1}^n x_i y_i = 0$, $A = \underline{(8)}$.
- $\int_0^1 \int_{y^2}^1 e^{\sqrt{x}} dx dy = \underline{(9)}$.

B. (14 points)

- The profit of buying a units of stock and b units of bond can be described by a function $W(a, b) = \sqrt{2}e^{-b} \cos a$. A profit-driven individual with a units of stock and b units of bond will move in the direction of maximum profit increase. Find the equation $b=f(a)$ for the path of a profit-seeking individual starting with $a=\pi/4$ and $b=0$.

C. (14 points)

- A bowl is in the shape of the graph of $z = x^2 + y^2$ from $z=0$ to $z=10$ inches. You plan to calibrate the bowl to make it into a rain gauge. Assume that the rain falls into the bowl vertically. Determine the height in the bowl which corresponds to 1 inch of rain.