國立臺灣大學95學年度碩士班招生考試試題

1. (30 points) Let $S: z = y^2 - x^2$, $p = (0,0,0) \in S$. Construct a normal vector field $\tilde{N}$ of S. Let $N : S \to S^2$ (S^2 is the standard 2-sphere) be the Gauss map induced by $\tilde{N}$. At the point p, compute the Gaussian curvature K and the mean curvature H by using the Gauss map N.

2. (30 points) Define the Christoffel symbols and compute them in terms of the coefficients of the first fundamental form.

3. (20 points) Let $S^2 : x^2 + y^2 + (z-1)^2 = 1$, $N = (0,0,2) \in S^2$. Consider the stereographic projection $\pi : S^2 \setminus \{N\} \to \mathbf{R}^2$ which carries a point $p = (x,y,z)$ of $S^2 \setminus \{N\}$ onto the intersection of the xy plane with the straight line which connects N to p.
i) Write down the map π explicitly.
ii) Prove that π is conformal, i.e. $||d\pi_p(v)||^2 = \lambda(p)||v||^2$ for all tangent vector v at p and for some positive function λ.

4. (20 points) Let $S \subset \mathbf{R}^3$ be a regular, compact, orientable surface which is not homeomorphic to a sphere. Prove that there exists at least one point on S where the Gaussian curvature is zero.