1. (7%)

Prove that if $p > 1$ and $x > 0$, $x^p - 1 \geq p(x-1)$.

2. (7%)

Find the point of the hyperbola $y^2 - \frac{x^2}{2} = 1$ that is nearest to $(x, y) = (0, 3)$, where the distance between a point (a, b) and the origin (0,0) is defined as $\sqrt{a^2 + b^2}$ in the 2-dimensional Euclidean space.

3. (7%)

Let $h(t) = \frac{7}{12}t^3 + t + 1$, $g(t) = t^3 - 2t^2 + 3t$, and $f(t) = (t-5) + \log_e t$, where $\log_e$ denotes the natural logarithm. Evaluate the second derivative of $f(g(h(x)))$ at $x = 0$.

4. (9%; 3% each)

Prove or disprove that the limit of each of the following functions exists when x approaches to zero, i.e., $\lim_{x \to 0} f(x)$.

4-a

$$f(x) = \frac{1}{x},\ x \in [-1, 1]$$

4-b

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ x^2 & \text{otherwise} \end{cases},\ x \in [-1, 1]$$

4-c

$$f(x) = \text{the integer part of } (1 + x),\ x \in [-1, 1]$$

5. (10%)

Suppose that $\lambda > 0$, please solve

$$\lim_{\lambda \to 0} \lambda \int_{\lambda^2}^{\lambda} \frac{\cos(x)}{x^{3/2}} dx.$$

[Hint] Application of the mean value theorem of the integral calculus: If $f(x)$ and $g(x)$ are continuous functions in $a \leq x \leq b$ and $g(x) \geq 0$, then $\int_a^b f(x)g(x)dx = f(\xi)\int_a^b g(x)dx$, where $a \leq \xi \leq b$.

6. (10 %)

If $\lim_{x \to a} \frac{2x^2 + bx + 3b}{2x - 2a} = 8$, then $(a, b) = ?$

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7. (10%)

If the inverse function of $f(x)$ is $f^{-1}(x) = \int_{\frac{2}{\pi}}^{\sqrt{x}} e^t \left(\frac{\sin(\frac{1}{t}) + t^2 \cos(\frac{1}{t})}{t^2} \right) dt$, then $f' \left[f^{-1} \left(\frac{16}{\pi^2} \right) \right] = ?$

8. (10%)

Calculate $\int_0^\infty e^{-\frac{1}{2}x^2} x \sin(2x) dx = ?$

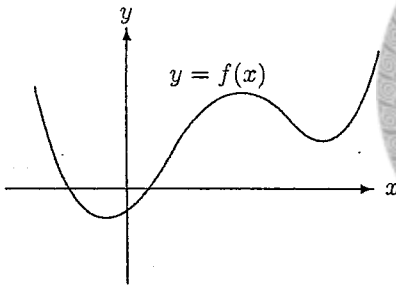
[**Hint**]: (1) $\int_{-\infty}^{\infty} e^{-x^2} = \sqrt{\pi}$; (2) Let $F(\xi) = \int_0^\infty e^{-\frac{1}{2}x^2} \cos(\xi x) dx$ and consider $F'(\xi)$

9. (10%)

Let $S_n = \frac{1}{\sqrt{n}} \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} \right)$. Then Find $\lim_{n \to \infty} S_n = ?$

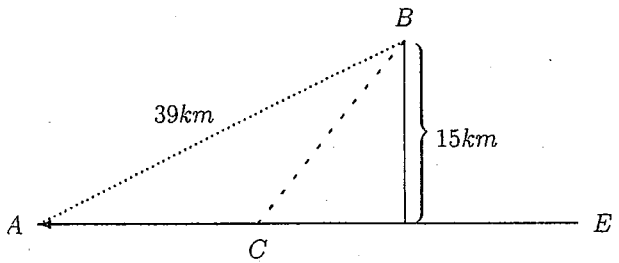
10. (10%)

Consider the plot of the function, $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, as follows. Please determine the signs of a, b, c, d, e.



11. (10%)

Suppose that the distance between two places A and B is 39 kilometers, and the distance of B to the road $\overline{AE}$ is 15 kilometers (see figure below). Now we want to construct a straight railway $\overline{BC}$ from B to a point C on the road $\overline{AE}$ in order to transport goods from A to B. Assume that the cost of transportation per kilometer on the railway is twice as much as the cost on the road. Please determine the optimal distance between A and C, which realizes the minimal cost!



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