國立臺灣大學96學年度碩士班招生考試試題



Notation : $Z$ is the set of all integers , $Q$ is the set of all rational numbers and $R$ is the set of all real numbers .

一 Let $D=\{3n \mid n\in Z\}$ be a subring of $Z$ and $I=\{15n \mid n\in Z\}$ , $J=\{9n \mid n\in Z\}$ are two ideals of D.

(1) Is $I$ a prime ideal ? Is the quotient ring $D/I$ a finite field? Prove your answer.

(2) Is $J$ a prime ideal ? Is the quotient ring $D/J$ a finite field? Prove your answer.

(3) Find all prime ideals of $D$.

二 Let $F$ be a finite field with order $|F|=m$ and we define that an element $\alpha\in F$ is called a multiple generator in $F$ if $F=\{0,1,\alpha,\alpha^2,\alpha^3,\cdots,\alpha^{m-2}\}$ . We have known that for any finite field $F$, there exist a multiple generator in $F$.

(1) If $\alpha$ is a multiple generator in $F$ then is $\alpha^{m-1}=1$ true? Prove your answer.

(2) If $F_{11}=\{0,1,2,\cdots,10\}$ is the finite field with order $|F_{11}|=11$ then find a multiple generator in $F_{11}$. How many multiple generators in $F_{11}$ ? Prove your answer.

(3) If $F_3=\{0,1,2\}$ is the finite field with order $|F_3|=3$ , $f(x)=x^2+2x+2$, $g(x)=x^2+x+1$ are polynomials in the polynomial ring $F_3[x]$ and $I=(f(x))$, $J=(g(x))$ are principal ideals of $F_3[x]$.

Is the quotient ring $F_3[x]/I$ a finite field ? If it is a finite field, find a multiple generator in $F_3[x]/I$ ? Prove your answer.

Is the quotient ring $F_3[x]/J$ a finite field ? If it is a finite field, find a multiple generator in $F_3[x]/J$ ? Prove your answer.

(4) If $F$ is the finite field with order $|F|=25$ then how many multiple generators in $F$ ? Prove your answer.

三 Let $i=\sqrt{-1}$, $u=7+8i$ and $v=2+i$. Find four integers $a,b,c,d\in Z$ such that $u=v(a+bi)+(c+di)$ and $c^2+d^2<5$.

四 Let $E$ be the splitting field of $x^4-3$ over $Q$ and $F$ be the splitting field of $x^4-4$ over $Q$.
Find the Galois group $Gal(E/Q)$ and the Galois group $Gal(F/Q)$.

五 Find the smallest positive integer $n$ such that $n>30$ and for any integer $a$, $a^n-a$ is divisible by 385.

六 Let $G$ be a group and $H=\{a^2 \mid a\in G\}$. If $H$ is a subgroup of $G$ then is $G$ an abelian group ?

Prove your answer.

試題隨卷繳回