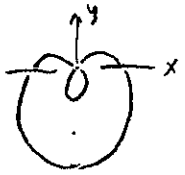
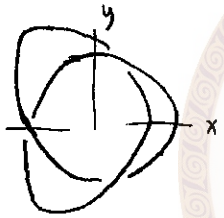
1. 

Let $x = r\cos\theta$, $y = r\sin\theta$ be expressed in polar coordinate. $r = 1 - 2\sin\theta$ is an immersed curve. $\theta = \pi/3$ and $2\pi/3$ are the same point, so-called self intersection. curvature $K(\theta = \frac{\pi}{3}) \neq K(\theta = \frac{2\pi}{3}) = ?$, Total curvature $\oint k\,ds = ?$ Here $\oint$ means $\int_{\theta=0}^{\theta=2\pi}$ and $ds = \sqrt{dx^2+dy^2}$. (25/100)

2. 

Let $R > 1$ be a radius,

$$\begin{cases} x = (R + \cos\varphi)\cos\theta \\ y = (R + \cos\varphi)\sin\theta \\ z = \sin\varphi \end{cases}$$ (25/100)

is a torus, $\varphi = 3t$, $\theta = 2t$ is a knot. When $t=0$ curvature $K(t=0) = ?$ Can you decide if the total curvature $\oint k\,ds > 4\pi$, $= 4\pi$ or $< 4\pi$? $\lim_{R\to\infty} \oint k\,ds = ?$

3. $\omega = (x^2 - y^2 - z^2)\,dx + 2xy\,dy + 2xz\,dz$ is a differential. Is ω a closed differential ? If not, can you solve the equation $\omega = 0$? If yes, its general solution $c = f(x, y, z) = ?$ (25/100)

4. In the unit disc $x^2 + y^2 < 1$, Poincaré metric $ds^2 = (dx^2 + dy^2)/(1 - x^2 - y^2)^2$. Let γ be an arc from $(0,0)$ to $(\frac{1}{2}, 0)$, $\inf_{\gamma} \int_{\gamma} ds = ?$ Let Γ be an arc from $(\frac{1}{2}, 0)$ to $(0, \frac{1}{2})$ $\inf_{\Gamma} \int_{\Gamma} ds = ?$

(25/100)

試題隨卷繳回