國立臺灣大學96學年度碩士班招生考試試題

1. (15 pts) Suppose that the random variable Y has a binomial distribution with n trials and success probability p, where n is a given constant and p is a $Beta(\alpha, \beta)$ random variable.

   (a) (7 pts) Find the conditional distribution of p given $Y = y$.

   (b) (8 pts) Determine the Bayes estimator of p under absolute error loss. (i.e. $E|\hat{p} - p|$)

2. (15 pts) Suppose that a single observation X is drawn from the following pdf:

$$f(x|\theta) = \begin{cases} 2(1-\theta)x + \theta & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

   where the value of θ is unknown ($0 \le \theta \le 2$). Suppose also that the following hypotheses are to be tested:

$$H_0 : \theta \le 1 \quad \text{versus} \quad H_1 : \theta > 1.$$

   (a) (10 pts) Derive the level α uniformly most powerful test.

   (b) (5 pts) Compute the power function of the test derived in (a).

3. (25 pts) Let $X_1, \ldots, X_n$, $n > 2$, be a random sample from the distribution with pdf

$$f(x|\theta) = \frac{1}{\theta} x^{1/\theta - 1}, \quad 0 < x < 1, \quad \theta > 0.$$

   (a) (10 pts) Show that $1/\bar{X}_n - 1$ is a consistent estimator of θ where $\bar{X}_n = \sum_{i=1}^{n} X_i / n$.

   (b) (5 pts) Obtain the maximum likelihood estimator of θ.

   (c) (10 pts) Derive the asymptotic distribution of the estimator in (b).

4. (45 pts) Suppose $Y_1, \ldots, Y_n, \ldots$ arise from the following $AR(1)$ model:

$$Y_j = \mu + \rho(Y_{j-1} - \mu) + \epsilon_j, j = 1, 2, \ldots$$

   where $\epsilon_j \overset{i.i.d.}{\sim} N(0, \sigma^2)$, $\mu \in R$, $\sigma^2 > 0$ and $\rho \in (-1, 1)$. When $Y_0 \sim N(0, \sigma^2/(1-\rho^2))$,

   (a) (7 pts) Determine $E(Y_i|Y_{i-1})$ and $E(Y_i)$.

   (b) (8 pts) Determine $Var(Y_i)$.

   (c) (10 pts) Write down the likelihood function of $\theta = (\mu, \sigma^2, \rho)$.

   (d) (10 pts) Derive the maximum likelihood estimate of θ.

   (e) (10 pts) Is $\bar{Y}$ a consistent estimator of μ? Give reason to support your answer

試題隨卷繳回