※ 注意：請於試卷上「非選擇題作答區」依序作答，並應註明作答之題號。

This exam consists of 10 multiple-choice questions. Each question worths 10 point. There is no penalty for guessing. However, students who only make correct choice but do not provide detailed and correct derivations and elaborations will not earn any point. Please write down your answer and derivations on the answer sheets.

(1) Let

$$a_n = n \sin\left(\frac{1}{n}\right) + (-1)^n \frac{\cos(n)}{n}$$

for n = 1,2,...,. Which statement is true of the sequence $\{a_n\}$?

A. It is bounded but does not converge.

B. It converges to 0.

C. It converges to a positive number.

D. It diverges to infinity.

E. It is unbounded and contains both arbitrarily large positive and arbitrarily large negative terms.

(2) Let $f : R \to R$ be a function with Taylor series converging to $f(x)$ for all real numbers x. If $f(0) = 2$, $f'(0) = 2$, and $f^{(n)}(0) = 3$ for $n \geq 2$, then $f(x) =$

A. $3e^x + 2x - 1$

B. $e^{3x} + 2x + 1$

C. $e^{3x} - x + 1$

D. $3e^x - x - 1$

E. $3e^x + 5x + 5$

(3) Let f be a function for which

$$I = \int_2^4 \int_x^{2x} f(x,y)dydx$$

exists. Which of the following expressions is equal to I with the order of integration reversed?

A. $\int_2^4 \int_2^y f(x,y)dxdy + \int_4^8 \int_{y/2}^4 f(x,y)dxdy$

B. $\int_x^{2x} \int_2^4 f(x,y)dxdy$

C. $\int_2^8 \int_{y/2}^y f(x,y)dxdy$

D. $\int_2^4 \int_{y/2}^4 f(x,y)dxdy + \int_4^8 \int_2^y f(x,y)dxdy$

E. $\int_2^4 \int_2^8 f(x,y)dxdy$

(4) Let $F(x)$ be a strictly decreasing continuously differentiable function on $[a,b]$. Then $\int_a^b |F'(x)|dx$ must equal to

A. $|F(b)| - |F(a)|$

B. $F(a) - F(b)$

C. $F(b) - F(a)$

D. $|F(a)| - |F(b)|$

E. $F(-b) - F(-a)$

(5) A population grows exponentially. At 10 years, the population is 1,000. At 20 years, it is 2,000. What was the approximate population at 5 years?

A. 140

B. 250

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C. 500

D. 700

E. 750

(6) A rectangle with one side lying along the x-axis is to be inscribed in the closed region of the xy-plane bounded by the lines $y = 0$, $y = 3x$, and $y = 30 - 2x$. What is the largest possible area of such a rectangle?

A. 135/8

B. 45

C. 135/2

D. 90

E. 270

(7) What is the cosine of the angle between the vectors $\begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$?

A. -3/4

B. 1/150

C. $3^{1/2}/15$

D. 1/2

E. 2

(8) The coordinates of an object moving through R^3 are $x = a\sin(t)$, $y = b\cos(t)$, and $z = \frac{1}{2}ct^2$, for time $t > 0$, where a, b, and c are constants. What is the speed of the object at time t?

A. $\sqrt{a^2\sin^2(t) + b^2\cos^2(t) + \frac{1}{4}c^2t^4}$

B. $\sqrt{a^2\cos^2(t) + b^2\sin^2(t) + 4c^2t^2}$

C. $\sqrt{a^2 + b^2 + c^2t^2}$

D. $\sqrt{a^2\cos^2(t) - b^2\sin^2(t) + 4c^2t^4}$

E. $\sqrt{a^2\cos^2(t) + b^2\sin^2(t) + c^2t^2}$

(9) Let $f(x) = e^{(x^3+x^2+x)}$ for any real number x, and let g be the inverse function for f. What is $g'(e^3)$?

A. $\frac{1}{34e^{39}}$

B. $\frac{1}{6e^3}$

C. $\frac{1}{6}$

D. 6

E. $6e^3$

(10) Which expression below is equal to

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{n}{(n+i)^2}$$

A. $\int_1^2 \frac{1}{x}dx$

B. $\int_0^1 \frac{1}{x^2}dx$

C. $\int_1^2 \frac{1}{x^2}dx$

D. $\int_{-1}^0 \frac{1}{x}dx$

E. $\int_2^3 \frac{1}{x^2}dx$

試題隨卷繳回