國立臺灣大學96學年度碩士班招生考試試題

※ 注意：請於試卷上依序作答，並應註明作答之大題及其題號。

1. (30 points)
   (a). Write an iterative pseudo algorithm that deletes all the leaves from a binary tree. What the time complexity of your algorithm?
   (b). A threaded tree is a tree where null pointers are replaced with pointers to their successor nodes. Write a pseudo algorithm to convert a binary search tree into a threaded binary search tree.

2. (20 points)
   Consider a connected undirected graph G=(V, E).
   (a) Write C++ declaration of the data structure using linked list that describes the graph.
   (b) Based on (a), write a C++ program that outputs the connected components (described by set of constituent nodes and edges) of the graph.

3. (20 points)
   In a round-robin tournament among n players, each player plays once against all other $n-1$ players. There are no draws, i.e., for a match between A and B, the result is either A beat B or B beat A.
   (a) Show that, after a round-robin tournament, it is always possible to arrange the n players in an order $p_1, p_2, \cdots, p_n$ such that p_1 beat p_2, p_2 beat p_3, $\cdots$, and p_{n-1} beat p_n.
   (b) Assume that the results of a round-robin tournament have been stored in an array. Design an algorithm that finds the order defined in (a). Please present your algorithm in an adequate pseudo code and make assumptions wherever necessary. Give an analysis of its time complexity. The more efficient your algorithm is, the more points you will be credited for this problem.

4. (20 points)
   You are required to solve, using the *dynamic programming* approach, the single-source shortest path problem, i.e., the problem of determining the shortest path from a designated start vertex to each of the other vertices, in an undirected weighted graph. As usual, you only need to compute the length of each shortest path (without recording the vertices on the path).
   (a) Define with suitable notations the needed recurrence relation, following the dynamic programming approach, that will serve as the basis of a solution algorithm. Please explain why (or why not) your solution allows edges with a negative weight.
   (b) Design an algorithm based on the recurrence relation in (a). Please present your algorithm in an adequate pseudo code and make assumptions wherever necessary. The more efficient your algorithm is, the more points you will be credited for this problem.

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5. (10 points)

(a) Please describe as precisely as possible two NP-complete problems that involve graphs. Argue why the problems are in NP.

(b) Assuming one of the two problems you have just described is NP-hard, prove that the other is also NP-hard.