第1題到第10題每題5分，請作答於「選擇題作答區」。第1題到第4題為單選；第5題到第10題為複選，需完全答對才有分數，答錯不倒扣。第11題到第14題，請作答於「非選擇題作答區」。

1. Which one of the following answers satisfies the differential equation $\frac{dy}{dx}=\frac{xy^2-\cos x\sin x}{y(1-x^2)}$ with the initial value y(0)=2. (A) $y(1-x^2)-\cos^2 x=1$, (B) $y^2(1-x^2)-\cos^2 x=3$, (C) $y^2(1-x)-\cos^2 x=3$, (D) $y(1-x^2)-\cos x\sin x=2$, (E) $y^2(1-x^2)-\cos x\sin x=4$.
2. The particular solution of the differential equation $y^{(4)}+y'''=1-x^2e^{-x}$ has the form of
   (A) $y_p=A+Bx^2e^{-x}+Cxe^{-x}+Ee^{-x}$, (B) $y_p=Ax+Bx^2e^{-x}+Cxe^{-x}+Ee^{-x}$,
   (C) $y_p=Ax^2+Bx^2e^{-x}+Cxe^{-x}+Ee^{-x}$, (D) $y_p=Ax^2+Bx^3e^{-x}+Cx^2e^{-x}+Exe^{-x}$, (E) none of above.
3. Let $F(s)$ be the Laplace transform of $f(t)$: (A) If $f(t)=e^{at}\cos\omega t$ then $F(s)=\frac{\omega}{(s-a)^2+\omega^2}$.
   (B) If $F(s)=\frac{1}{s^n}$ $n=1,2,3\ldots$, then $f(t)=\frac{t^{n-1}}{(n-1)!}$. (C) The Laplace transform of $\frac{df(t)}{dt}$ is $sF(s)$. (D) The inverse Laplace transform of $\frac{F(s)}{s}$ is $\int f(t)dt$. (E) none of above.
4. We are going to solve the differential equation system: $\frac{dx}{dt}=\frac{1}{2}x$ and $\frac{dy}{dt}=x-\frac{1}{2}y$ with boundary conditions x(0)=3 and y(0)=5. The solution has following forms: $x=c_1e^{\lambda_1 t}+c_2e^{\lambda_2 t}$ and $y=c_3e^{\lambda_1 t}+c_4e^{\lambda_2 t}$ with $\lambda_1<\lambda_2$. $c_1\sim c_4$ and λ_1,λ_2 are constants. Which one of the following statements is correct?
   (A) $\lambda_1+\lambda_2=1$ (B) $c_2=c_4$ (C) $c_1=-c_3$ (D) $c_2+c_3=4$ (E) none of above.
5. What statements in the following are correct?
   (A) $\frac{dy}{dx}+P(x)y=f(x)y^n$ is a linear differential equation for n=0.
   (B) $\frac{dy}{dx}+P(x)y=f(x)y^n$ is a linear differential equation for n=1.
   (C) $\frac{dy}{dx}+P(x)y=f(x)y^n$ is a linear differential equation for n=2.
   (D) $\frac{dy}{dx}+P(x)y=f(x)y^n$ cannot be reduced to a linear differential equation for n $\neq$ integer.
   (E) $\frac{dy}{dx}+P(x)y=f(x)y^n$ can be reduced to a linear differential equation for n > 4.
6. For the differential equation $ax^2y''+bxy'+cy=0$ with *a*, *b*, and *c* real numbers,
   (A) the solution always contains this term px^m, where *p* and *m* are real numbers.
   (B) if *a* = 1, *b* = -2, *c* = -4, then the solution has the form of $y=px^m+qx^n$, where *m* and *n* are integers; *p* and *q* are real numbers.
   (C) if *a* = 4, *b* = 8, *c* = 1, then the solution has the form of $y=px^m+qx^n$, where *m* and *n* are integers; *p* and *q* are real numbers.
   (D) if *a* = 4, *b* = 0, *c* = 17, then the solution is $y=x^{1/2}[p\cos(2\ln x)+q\sin(2\ln x)]$, where *p* and *q* are real numbers.
   (E) if *a* = 1, *b* = -3, *c* = 3, then the solution has the form of $y=px^m+qx^n$, where *m* and *n* are integers; *p* and *q* are real numbers.
7. Consider the differential equation $(x-1)y''+y'=0$.
   (A) There exist two independent power series solutions centered at 0, both of them having the radius of convergence 1.
   (B) One of the solution is $c\sum_{k=1}^{\infty}\frac{x^k}{k}$ with the radius of convergence 1.

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(C) Let ϕ be the solution associated with the initial conditions $y(0)=0$ and $y'(0)=5$, then $\phi=5x$.
(D) Let ϕ be the solution associated with the initial conditions $y(0)=5$ and $y'(0)=0$, then $\phi=5$.
(E) none of above

8. Let $L(\ )$ and $L^{-1}(\ )$ be the Laplace and the inverse Laplace transforms; respectively.
(A) $L^{-1}(c_1F(s)+c_2G(s))=c_1L^{-1}(F(s))+c_2L^{-1}(G(s))$.
(B) $L^{-1}((\frac{2}{s}-\frac{1}{s^3})^2)=4t-\frac{2}{3}t^3+t^5$.
(C) $L((\cos t)^2)=\frac{1}{2}(\frac{1}{s}+\frac{s}{s^2+4})$.
(D) $L^{-1}(e^{-st_0})=\delta(t-t_0)$, where $\delta(\ )$ is the Dirac Delta function.
(E) none of above

9. If $f(x)=x^2+x$ for $0<x<2$, we are going to expand $f(x)$ in cosine series, sine series and Fourier series. Please find the converged value for different series:
(A) $f(2)=6$ for cosine series, (B) $f(2)=6$ for sine series, (C) $f(-1)=2$ for cosine series,
(D) $f(-1)=0$ for Fourier series, (E) $f(-5)=2$ for sine series.

10. A thin rectangular plate coincides with the region defined by $0\le x\le 5$ and $0\le y\le 3$. The right and top ends of the plate are insulated. The left end of the plate is kept at 0 degree and bottom end is held at temperature $f(x)$. Find the suitable differential equation and boundary conditions for the steady-state temperature $u(x,y)$:
(A) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, $0<x<5$ and $0<y<3$, (B) $\left.\frac{\partial u}{\partial x}\right|_{x=5}=0$ for $0<y<3$, (C) $\left.\frac{\partial u}{\partial y}\right|_{y=3}=0$ for $0<x<5$,
(D) $u(5,y)=0$ for $0<y<3$, (E) $u(x,0)=f(x)$ for $0<x<5$.

11. For any matrix A, let $\mathcal{N}(A)$ denote its null space. In the real space $\Re^n$, consider the inner product $\langle \mathbf{x},\mathbf{y}\rangle=x_1y_1+\cdots+x_ny_n$ and 2-norm $\|\mathbf{x}\|=\langle \mathbf{x},\mathbf{x}\rangle^{1/2}$ for every vectors $\mathbf{x}=[\,x_1\cdots x_n\,]^T$ and $\mathbf{y}=[\,y_1\cdots y_n\,]^T$ in $\Re^n$. Suppose S is a subspace of $\Re^n$. Let $S^\perp$ be the orthogonal complement of S in $\Re^n$. For the following matrix A, (a) find a basis β for $\mathcal{N}(A)^\perp$, and (b) check if $\mathbf{x}=[\,0\ 0\ 1\ 0\ 0\,]^T$ is a vector with the smallest 2-norm satisfying $A\mathbf{x}=[\,1\ 2\ 2\ 1\,]^T$ and explain why. (15%)

$$A=\begin{bmatrix}3&3&1&3&3\\2&4&2&4&2\\0&3&2&3&0\\-1&1&1&1&-1\end{bmatrix}$$

12. $X=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$. $X^{50}=?$ (10%)

13. Use Gaussian elimination procedures to find the reduced row echelon form, rank, and nullity of the matrix below: (10%)

$$\begin{bmatrix}1&0&-2&-1&0&-1\\2&-1&-6&-2&0&-4\\0&1&2&1&1&1\\-1&2&6&3&1&2\end{bmatrix}$$

14. Find an orthogonal basis for the subspace $C([0,1])$ that is spanned by $\{1,e^t,e^{-t}\}$. (15%)
Note: The definition of an inner product for f and g in $C([a,b])$ is
$\langle f,g\rangle=\int_a^b f(t)\,g(t)\,dt$

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