※ 注意: 請於試卷上「選擇題作答區」依序作答。
選擇題 (單選; 每題答對得 2 分; 答錯或未答得 0 分)

1. In the questions below suppose that $Q(x)$ is "$x + 1 = 2x$", where x is a real number. Let y be a natural number. Which of the following is false?
(A) $\forall y \exists x\, Q(x+y)$, (B) $\exists x \neq y (Q(x^2) \wedge Q(y^2))$, (C) $\exists x Q(x)$,. (D) $\forall y \exists x\, Q(x*y+1)$, (E) $\forall x \exists y\, Q(x*y)$.
2. In the questions below $P(x,y)$ means "$x + 2y = xy$", where x and y are integers. Which of the following is true?
(A) $\forall x \exists y P(x,y)$, (B) $\exists x \forall y P(x,y)$, (C) $\forall y \exists x P(x,y)$, (D) $\exists y P(3,y)$, (E) $\exists y \forall x P(x,y)$.
3. In the question below suppose the variable x represents students, y represents courses. Which statement is different from the other four?
(A) $\exists x \forall y \neg T(x,y)$, (B) $\neg \forall x \exists y \neg T(x,y)$, (C) $\neg \forall x \neg \forall y \neg T(x,y)$, (D)$\neg \forall x \exists y T(x,y)$, (E) $\exists x \neg \exists y T(x,y)$
4. Again, suppose the variable x represents students, y represents courses. Which statement asserts "No course is being taken by all students"?
(A) $\forall y \exists x T(x,y)$, (B) $\exists y \forall x T(x,y)$, (C) $\neg \exists y \forall x T(x,y)$, (D) $\neg \exists x \exists y T(x,y)$, (E) $\exists y \forall x \neg T(x,y)$
5. Determine which of the given set is not the power set of some set..
(A)$\{\varnothing,\{\varnothing\},\{a\},\{\{a\}\},\{\{\{a\}\}\},\{\varnothing,a\},\{\varnothing,\{a\}\},\{\varnothing,\{\{a\}\}\},\{a,\{a\}\},\{a,\{\{a\}\}\},\{\{a\},\{\{a\}\}\}\}$,
(B)$\{\varnothing,\{a\}\}$, (C) $\{\varnothing,\{\{a\}\}\}$, (D) $\{\varnothing,\{a\},\{\varnothing,a\}\}$, (E) $\{\varnothing,\{a\},\{\varnothing\},\{a,\varnothing\}\}$
6. Suppose $A = \{1,2,3,4,5\}$, and P(A) denote the power set of A. Determine which statement is false.
(A) $\{\{3\}\} \subseteq P(\text{A})$, (B) $\varnothing \subseteq A$, (C) $\{\varnothing\} \subseteq P(A)$, (D) $\varnothing \subseteq P(\text{A})$, (E) $\{2,4\} \in A \times A$
7. Which of the following is true?
(A) $A-(B-C)=(A-B)-C$, (B) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,
(C) $(A-C)-(B-C)=A-B$, (D) $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$,
(E) If $A \cup C = B \cup C$, then $A = B$
8. Which of the following rule describes a function with the given domain and codomain? (Note that N, Z, and R denote the set of natural numbers, integers and real numbers, respectively.)
(A) $f : \mathbf{N} \to \mathbf{N}$ where $f(n) = \sqrt{n}$, (B) $h : \mathbf{R} \to \mathbf{R}$ where $h(x) = \sqrt{x}$,
(C) $F : \mathbf{R} \to \mathbf{R}$ where $F(x) = \dfrac{1}{x-5}$, (D) $F : \mathbf{Z} \to \mathbf{Z}$ where $F(x) = \dfrac{1}{x^2-5}$,
(E) $g : \mathbf{N} \to \mathbf{N}$ where $g(n)$ = any integer $> n$
9. How many of the following functions have inverse?
$f : \mathbf{Z} \to \mathbf{Z}$ where $f(x) = x \bmod 10$
$f : A \to B$ where $A = \{a,b,c\}$, $B = \{1,2,3\}$ and $f = \{(a,2),(b,1),(c,3)\}$
$f : \mathbf{R} \to \mathbf{R}$ where $f(x) = 3x - 5$
$f : \mathbf{R} \to \mathbf{R}$ where $f(x) = \lfloor 2x \rfloor$.
(A) 0, (B) 1, (C) 2, (D) 3, (E) 4
10. Which of the following grows faster than the others? (The base for the logarithmic functions is assumed to be 2.)
(A) $O(n \cdot \log n)$, (B) $O(\log n!)$, (C) $O(\log n^n)$,
(D) $O(n \cdot 2^{\log \log n})$, (E) $O(n^{1.001})$
11. How many positive integers less than 30 that are relatively prime to 20?
(A) 10, (B) 11, (C) 12, (D)13, (E) 14
12. Which of the following is false?
(A) If $a \equiv b \pmod{m}$, then $2a \equiv 2b \pmod{m}$,
(B) If $a \equiv b \pmod{2m}$, then $a \equiv b \pmod{m}$,
(C) If $a \equiv b \pmod{m}$, then $2a \equiv 2b \pmod{2m}$,
(D) If $a \equiv b \pmod{m^2}$, then $a \equiv b \pmod{m}$,
(E) If $a \equiv b \pmod{m}$, then $a \equiv b \pmod{2m}$
13. Consider all bit strings of length 12. Which of the following statement is not true?
(A) There are 2^9 strings begin with 110.,
(B) There are 2^8 strings begin with 11 and end with 10,
(C) There are $\binom{12}{4} \cdot 2^8$ .strings have at most four 1s,
(D) There are $2 \cdot 2^{10} - 2^8$ strings begin with 11 or end with 10,
(E) There are $\binom{12}{4}$ strings have exactly four 1s.

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14. A factory makes automobile parts. Each part has a code consisting of a digit, a letter, and a digit, with the digits distinct, such as 5C7, 1O6, or 3Z0. Last week the factory made 8,000 parts. Find the minimum number of parts that must have the same serial number.
(A) 1, (B) 2, (C) 3, (D)4, (E) 5
15. A professor teaching a Discrete Math course gives a multiple choice quiz that has four questions, each with four possible responses: a, b, c, d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)?
(A) 65, (B) 129, (C) 257, (D) 513, (E) 1025
16. How many permutations of 12345 are there that leave 3 in the third position but leave no other integer in its own position??
(A) 8, (B) 9, (C) 10, (D) 12, (E) 15
17. Suppose you and a friend each choose at random an integer between 1 and 8. For example, some possibilities are (3,7), (7,3), (4,4), (8,1), where your number is written first and your friend's number second. Which of the following has the largest probability?
(A) p(your number is greater than your friend's number),
(B) p(sum of the two numbers picked is < 4),
(C) p(both numbers match),
(D) p(you pick 5 and your friend picks 8),
(E) p(the sum of the two numbers is a prime)
18. Consider an experiment consists of picking at random a bit string of length five. Consider the following events:
E_1: the bit string chosen begins with 1;
E_2: the bit string chosen ends with 1;
E_3: the bit string chosen has exactly three 1s.
Which of the following probability is unique (different from the other four)?
(A) $p(E_1 \mid E_2)$, (B) $p(E_2 \mid E_3)$, (C) $p(E_3 \mid E_2)$, (D) $p(E_1 \mid E_3)$, (E) $p(E_3 \mid E_1 \cap E_2)$
19. Determine which of the following recurrence relation is not a linear homogeneous recurrence relation with constant coefficients.?
(A) $a_n = a_{n-1} + 1$, (B) $a_n = ca_{n-1}$, (C) $a_n = a_{n-3}$,
(D) $a_n - 7a_{n-2} + a_{n-5} = 0$, (E) $a_n = 0.7a_{n-1} - 0.3a_{n-2}$
20. The solutions to $a_n = -3a_{n-1} + 18a_{n-2}$ have the form $a_n = c \cdot 3^n + d \cdot (-6)^n$. Which of the following is not a solutions to the given recurrence relation?
(A) $a_n = 3^{n+1} + (-6)^n$, (B) $a_n = 5(-6)^n$, (C) $a_n = 3^n + 6^n$,
(D) $a_n = 3^{n-2}$, (E) $a_n = \pi(3^n + (-6)^n)$.
21. Consider the recurrence relation $a_n = 2a_{n-1} + 3n$. Which of the following is false?
(A) The associated homogeneous recurrence relation is $a_n = 2a_{n-1}$,
(B) The general solution to the associated homogeneous recurrence is $a_n = c2^n$,
(C) $a_n = -3n - 6$ is a particular solution to the given recurrence relation,
(D) The general solution to the given recurrence relation is $a_n = -3n - 6 + c2^n$,
(E) $a_n = -3n - 6 + 7 \cdot 2^n$ is the particular solution to the given recurrence relation when $a_0 = 0$.
22. Consider the power series of each of the following functions. Which has the smallest coefficient of x^8?
(A) $(1+x^2+x^4)^3$, (B) $(1+x^2+x^4+x^6)^3$, (C) $(1+x^2+x^4+x^6+x^8+x^{10})^3$,
(D) $(1+x)(1+x^2)(1+x^3)(1+x^4)(1+x^5)$, (E) $(1+x^2+x^4+x^6+x^8)^3$
23. What is the number of ways in which nine identical blocks can be given to four children, if the oldest child gets either 2 or 3 blocks.
(A) 32, (B) 64, (C) 80, (D) 96, (E) 128
24. What is the number of positive integers ≤ 1000 that are multiples of at least one of 3,5,11?
(A) 500, (B) 505, (C) 510, (D) 515, (E) 520
25. What is the number of strings of 0s, 1s, and 2s of length six that have no consecutive 0s.?
(A) 448, (B) 450, (C) 452, (D) 454, (E) 456

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26. Given a set A such that $|A| = n$ (i.e., A has n elements). What is the number of binary relations on A?
(A) n^2, (B) $2n$, (C) 2^{n^2}, (D) 2^n, (E) n^3

27. Suppose $A = \{2,3,6,9,10,12,14,18,20\}$ and R is the partial order relation defined on A where xRy means x is a divisor of y. (For example, *2R6 and 3R12*.) What is $lub(\{2,9\})$?
(A) 9 (B) 1 (C) 18 (D) 2 (E) Does not exist.

28. What is the smallest equivalence relation on {1,2,3} that contains (1,2) and (2,3)?
(A) {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)},
(B) {(1,1),(1,2),(1,3), (2,2),(2,3),(3,2),(3,3)}, (C) {(1,2), (2,3),(1,3)},
(D){(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)} (E) {(1,2), (2,3)}

29. What is the smallest partial order relation on {1,2,3} that contains (1,1), (3,2), (1,3)?
(A) {(1,1), (3,2),(1,3)} (B) {(1,1),(2,2),(3,3),(3,2),(1,3),(1,2)} (C) {(1,1),(2,2),(3,3),(3,2),(1,3)}
(D) {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)} (E) None of the above (以上皆非)

30. Let R be the relation on A={1,2,3,4,5} where
R= {(1,1), (1,3), (1,4), (2,2), (3,1), (3,3), (3,4), (4,1), (4,3), (4,4), (5,5)}. What are the equivalence classes for the partition of A given by R? (A) {1,4},{3,2},{5} (B){1,2,3,4},{5} (C) {1,3,4},{2},{5}
(D) {1},{2},{3},{4},{5} (E) {1,2,3,4,5}

31. Suppose the relation R is defined on the set $\mathbf{Z}$ where aRb means that $ab \le 0$. ($\mathbf{Z}$ is the set of all integers.) Which of the following statements is true? (A) R is an equivalent relation, (B) R is reflexive, symmetric but not transitive, (C) R is anti-symmetric, transitive, but not reflexive, (D) R is not reflexive, not transitive (E) R is a partial order relation,

32. Suppose R and S are relations on $\{a,b,c,d\}$, where $R = \{(a,b),(a,d),(b,c),(c,c),(d,a)\}$ and $S = \{(a,c),(b,d),(d,a)\}$. We define $R \circ S = \{(x,z) \mid \exists y: xSy \wedge yRz\}$. What is $R \circ S$?
(A) $\{(a,a),(b,b),(c,c),(d,d)\}$, (B)$\{(a,c),(b,a),(d,b),(d,d)\}$, (C) $\{(a,c),(b,a),(b,d),(d,d)\}$,
(D) $\{(a,a),(a,c),(b,a),(d,b),(d,d)\}$, (E) None of the above (以上皆非)

33. Let $K_{m,n}$ be the complete bipartite graph with m and n vertices on the two sides of the graph. What is the number of edges in $K_{m,n}$? (A) $m+n$ (B) mn (C) m^n (D) $m(n-1)/2$ (E) $(m-1)(n-1)$

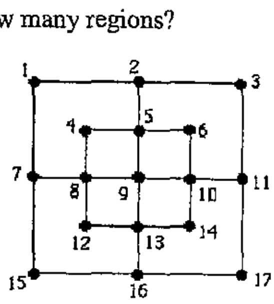
34. What is the largest value of n for which K_n is planar? (Note that K_n is the n-vertex complete graph.)
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

35. What is the largest value of n for which $K_{6,n}$ is planar? (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

36. If G is a planar connected graph with 20 vertices, each of degree 3, then G has how many regions?
(A) 9 (B) 10 (C) 12 (D) 14 (E) 15

37. Consider the graph G shown on the right. Which of the following is true?
(A) G has an Euler circuit but no Hamilton circuit.
(B) G has an Euler circuit and a Hamilton circuit.
(C) G has a Hamilton circuit but no Euler circuit.
(D) G has no Hamilton circuit and no Euler circuit.
(E) None of the above (以上皆非)

Graph G



38. How many non-isomorphic trees with four vertices exist? (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

39. What is value of the arithmetic expression whose prefix representation is − 5 / * 6 2 − 5 3 (where −, / and * denote integer subtraction, division and multiplication, respectively)?
(A) 1 (B) -1 (C) 2 (D) 0 (E) 3

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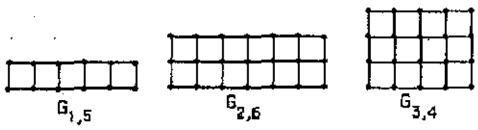
40. The *grid graph* $G_{m,n}$ refers to the graph obtained by taking an $m \times n$ rectangular grid of streets ($m \le n$) with m north/south blocks and n east/west blocks. See the right for some examples.



What is the vertex-chromatic number for $G_{m,n}$? (A) 2 (B) 3 (C) 4 (D) $m+n$ (E) mn

41. Again consider $G_{m,n}$. What is the number of edges of $G_{m,n}$?
(A) $(m+1)(n+1)$ (B) $4mn$ (C) $n(m+1)+m(n+1)$ (D) m^n (E) *None of the above* (以上皆非).

42. Suppose H is a graph with vertices a, b, c, d, e, f with adjacency matrix shown on the right. What is distance between vertex a and vertex c? (By distance we mean the number of edges along the shortest path between a and c.)
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

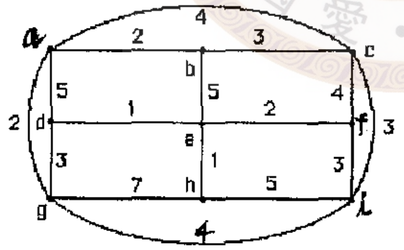
43. Suppose you have 5 coins, one of which is counterfeit (either heavier or lighter than the other four). You use a pan balance scale to find the bad coin and determine whether it is heavier or lighter. What is the minimum number of weighings needed to guarantee that you find the bad coin and determine whether it is heavier or lighter? (A) 2 (B) 3 (C) 4 (D) 5 (E) more than 5

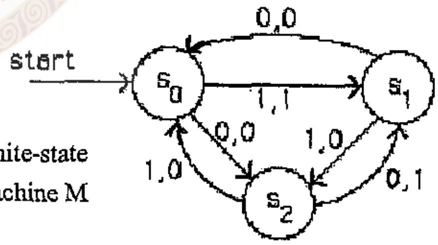
44. The string $- * 2 - x\ a + 4\ y$ is prefix notation for an algebraic expression. What is the expression in postfix notation? (A) $2 * x - a - + 4\ y$ (B) $2 * x - a - 4 + y$ (C) $2\ x\ a - * 4\ y + -$ (D) $2\ x - a * 4\ y + -$
(E) None of the above (以上皆非)

45. Every full binary tree with 50 leaves has how many vertices? (A) 49 (B) 50 (C) 51 (D) 99 (E) 100

46. What is the weight of the minimum spanning tree of the graph H shown below?
(A) 15 (B) 17 (C) 19 (D) 21 (E) 23

Graph H





Finite-state machine M

47. What is the output produced by the above finite-state machine M when the input string is 11101?
(A) 10100 (B) 10010 (C) 11101 (D) 10101 (E) 10000

48. Let $A = \{1, 10\}$ and consider five strings λ, 1101, 1110111, 10110, 110011. (λ is the empty string.) How many of the above five strings belong to A^*? (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

49. Which of the following phrase-structure grammars (where S is the start symbol) produces $\{1^{2n} \mid n > 0\}$?
(A) $S \to S11, S \to 11A, A \to 11$ (B) $S \to S11, S \to 11, A \to 1$ (C) $S \to S11, S \to A, S \to A1, A \to 1$
(D) $S \to S11, S \to AA, A \to 11$ (E) None of the above (以上皆非)

50. Consider relation R with its matrix representation shown on the right. R is:
(A) reflexive and symmetric (B) reflexive and anti-symmetric
(C) not transitive and not anti-symmetric (D) symmetric and transitive
(E) None of the above (以上皆非)

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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