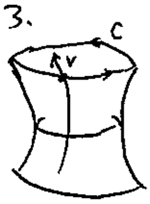
1、 In cylindrical coordinate, $x = r\cos\theta$, $y = r\sin\theta$, $z = z$. The surface $z = 2\theta$ is like a helicoid. At the point $(x, y, z) = (0, 0, \pi/2)$ mean curvature $H = ?$, Is it a minimal surface? (25/100)

2. A curve in cylindrical coordinate $C: (r, \theta, z) = (t, t, t)$

$t > \pi/2$. Can you find a point on C where the osculating plane is horizontal, i.e. parallel to the x-y plane? If yes $t = ?$ (25/100)

3.



$x^2 + y^2 - z^2 = 1$ is a hyperboloid, and $C: \{z = 1\}$ is a curve on it. $\vec{V} = (1, 0, \sqrt{2})$ is a tangent vector to the hyperboloid at $(x, y, z) = (\sqrt{2}, 0, 1)$. Parallel translate $\vec{V}$ along C counterclockwise and back to the point $(\sqrt{2}, 0, 1)$ do you get the same vector $\vec{V}$? If not, new vector $= (?, ?, ?)$ (25/100)

4. $L_1 = \{x = 0 = y\}$, $L_2 = \{x - 1 = 0 = z\}$, $X = \mathbb{R}^3 - L_1 - L_2$. Can you find a covering space $p: \tilde{X} \to X$ so that the fundamental group $\pi_1(\tilde{X})$ is isomorphic to $\mathbb{Z}$? If yes, is $p: \tilde{X} \to X$ a finitely-sheeted covering? If yes, find the number of sheets. If infinite, determine whether it is countable or un-countably infinite. (25/100)

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