1. [3%] (a) Answer True or False. Floating-point numbers are distributed uniformly throughout their range.
[3%] (b) Answer True or False. An iterative method converges in second order provided the errors at successive iteration are $10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}, \ldots$.
[3%] (c) Answer True or False. Newton's method for solving $f(x)=0$ can be interpreted as approximating the function f near the current iterate x_k by the tangent line at $f(x_k)$ and always converges quadratically.
[4%] (d) Give an example to show that floating point addition is not necessrily associative.
[4%] (e) List tree sources of error in scientific computation.
[4%] (f) Give an example to explain the distinction between truncation error and rounding error.

2. [10%] Explain how you may choose x_i and ω_i for $i=1,\ldots,n$ such that the weighted quadrature approximation $\int_0^1 \frac{f(x)}{\sqrt{x}}dx \approx \sum_{i=1}^{n}\omega_i f(x_i)$ has no error for polynomials $f(x)$ of as large a degree as possible.

3. [15%] Calculate the natural cubic spline interpolating the points $(1,1)$, $(2,\frac{1}{2})$, $(3,\frac{1}{3})$, and $(4,\frac{1}{4})$.

4. Consider a tridiagonal linear system $Ax=f$, where

$$A=\begin{bmatrix} b_1 & c_1 & 0 & 0 & \cdots & 0 \\ a_2 & b_2 & c_2 & 0 & \cdots & 0 \\ 0 & a_3 & b_3 & c_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & a_{n-1} & b_{n-1} & c_{n-1} \\ 0 & \cdots & \cdots & 0 & a_n & b_n \end{bmatrix} \text{ is nonsingular, } x=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}, \text{ and } f=\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_{n-1} \\ f_n \end{bmatrix}.$$

[6%] (a) Find the general form of the LU factorization of matrix A. Here the matrix L is a unit lower triangular matrix and the matrix U is an upper triangular matrix.
[6%] (b) Write a pseudocode to construct the matrix L and U.
[6%] (c) Analyze the arithmetic computational cost of LU factorization in terms of n.
[6%] (d) Take A and f as inputs, write a pseudocode to solve the linear system $Ax=f$ by using the pseudocode in part (b) in this problem.

5. Consider the following one dimensional boundary value problem $u''(x)=f(x)$, for $x\in(0,1)$ and $u(0)=u(1)=0$. We first discretize the domain $(0,1)$ by n uniformly distributed grid points and then solve the problem by a finite difference method.
[8%] (a) Using the Taylor series to derive a central second-order finite difference approximation of $u''(x)$.
[7%] (b) Use the notation $u_j \approx u(x_j)$, for $j=1,\ldots,n$, to write the four finite difference equations for $u(x)$ at $x=0.2$, 0.4, 0.6, and 0.8.
[15%] (c) Write a pseudocode to implement the finite difference method for the equation. You may use your pseudocodes in Problem 4.

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