Notice: You *must* show all your *work* in order to receive *full* credit!

(1) (15 points) Let the subspace V be generated by

$$v_1 = (2,0,0,8)),\quad v_2 = (0,3,0,1),\quad v_3 = (2,0,1,2),$$

and let the subspace W be generated by

$$w_1 = (1,2,3,4),\ \ w_2 = (1,5,0,-3),\ \ w_3 = (1,0,5,4).$$

Find a basis for $V \cap W$.

(2) (20 points) Find the *Jordan canonical form* of the matrix:

$$\begin{bmatrix} 3 & 1 & 1 & 2 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

(3) (15 points) Let $u_i = t \cdot v_i - w_i$, for $i = 1,2,3,4,5$, where

$$v_1 = (1,0,0,0,0), v_2 = (2,1,0,0,0), v_3 = (3,2,1,0,0), v_4 = (4,3,2,1,0),$$
$$v_5 = (5,4,3,2,1),\qquad w_1 = (55,40,26,14,5),\ \ w_2 = (40,30,20,11,4),$$
$$w_3 = (26,20,14,8,3),\ \ w_4 = (14,11,8,5,2),\qquad w_5 = (5,4,3,2,1).$$

Show that if $t \neq 1$ then u_1, u_2, u_3, u_4, u_5 form a *basis* of $\mathbb{R}^4$.

(4) (15 points) Find a matrix $Q \in M_n(\mathbb{Q})$, which is a matrix with *rational* entries, so that $Q \sim P$ where P is the *complex* matrix:

$$\begin{bmatrix} 0 & 0 & 0 & \frac{1-i}{\sqrt{2}} \\ \frac{1+i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & \frac{-1+i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{-1-i}{\sqrt{2}} & 0 \end{bmatrix}$$

Here $A \sim B$ means A and B are *similar* in the sense that $A = X^{-1}BX$ for some invertible square matrix X.

(5) (15 points) Let A and B be two $n \times n$ matrices such that $AB = 0$. Show that the 0-eigen space of BA has dimension at least $n/2$.

(6) (20 points) Let $V = \{f(x) \mid f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4,\ a_0, \ldots a_4 \in \mathbb{R}\}$.

Claim: For every *linear functional* $\Phi : V \longrightarrow \mathbb{R}$, there exist a polynomial $g_\Phi(x) \in V$ such that

$$\Phi(f(x)) = \int_0^1 f(x) g_\Phi(x)dx, \quad \text{for every } f(x) \in V.$$

Prove or disprove the Claim.

試題隨卷繳回