1. (25 pts) Let Z denote a continuous random variables with a uniform distribution in the interval $[-1, 1]$, and let X denote a discrete random variable, independent of Z, taking values 1 and 2 with probabilities p_1 and p_2, respectively ($p_1 + p_2 = 1$). Define a random variable Y as

$$Y = \begin{cases} Z & \text{if } X = 1, \\ -Z & \text{if } X = 2. \end{cases}$$

(a) (10 pts) Find the cumulative distribution function of Y.

(b) (5 pts) Find $E(Y)$ and $E(Y^2)$.

(c) (5 pts) Show that the correlation between Y and Z is $p_1 - p_2$.

(d) (5 pts) When $p_1 = p_2$, are Y and Z independent? Give reason to justify your claim.

2. (25 pts) Let $X_1, X_2, \ldots, n > 2$, be a random sample from the distribution with pdf

$$f(x|\theta) = \begin{cases} \frac{1}{\theta^2}(x+\theta), & -\theta \le x \le 0 \\ -\frac{1}{\theta^2}(x-\theta), & 0 \le x \le \theta \end{cases}$$

where $\theta > 0$.

(a) (10 pts) Find $E(X)$ and $E(X^2)$.

(b) (8 pts) Use the method of moments to give an estimator of θ, $\hat{\theta}_{MM}$.

(c) (7 pts) Show that $\hat{\theta}_{MM}$ is a consistent estimator of θ. (You can use Law of Large Numbers to answer this question.) If you are not sure whether the estimator you derived in (b) is correct, you can work on (c) assuming $\hat{\theta}_{MM} = \sqrt{\Sigma_{i=1}^{n}(X_i - \bar{X})^2/n}$ where $\bar{X}$ is the sample average.

3. (25 pts) Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean θ and variance 1. It is known that $\theta \ge 0$.

(a) (10 pts) Derive the maximum likelihood estimator of θ_n, $\hat{\theta}$.

(b) (10 pts) When $\theta = 0$, derive the distribution of $\hat{\theta}_n$.

(c) (5 pts) When $\theta = 1$, derive the distribution of $\hat{\theta}_n$ as n goes to infinity.

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4. (25 pts) Someone doubts that the probability of getting *Head* for a particular coin may be higher than 0.5. Let p denote the probability of getting *Head* for that particular coin. Statistician NP proposes the following testing procedure:

NP1. $H_0 : p = 0.5$ versus $H_a : p = 0.6$.

NP2. Set the rejection region to be $\{X \geq c\}$ where X denotes the number of *Head* when the coin is flipped ten times.

NP3. Determine c by setting the probability of committing Type I error to be 0.1.

(a) (9 pts) Determine c for Statistician NP.

(b) (9 pts) For c derived in (a), determine the probability of committing Type II error.

(c) (7 pts) Is it possible for you to come up a better procedure than the one proposed by Statistician NP with smaller probability of committing Type II error and the probability of committing Type I error is no more than 0.1? Give an argument to support your answer.

In answering this question, you can use the following table which gives $P(X \leq k)$.

	0	1	2	3	4	5	6	7	8	9	10
$p = 0.5$	.001	.011	.055	.172	.377	.623	.828	.945	.989	.999	1
$p = 0.6$	.000	.002	.012	.055	.166	.367	.618	.833	.954	.995	1