國立臺灣大學98學年度碩士班招生考試試題

(1)-(10)題各 8 分，按題序標清題號寫下答案，其他計算式一律不予計分。

1. Let $y = y(x)$ satisfy $y(1) = 1$ and $x^3 + 2x^2y - y^3 = 2$. Find $y''(2)$.

2. Find $\dfrac{d}{dx} 2^{\tan x}\Big|_{x=\pi/4}$.

3. Find $\lim\limits_{x\to\infty} (3x^2+5)^{\frac{1}{\ln x+1}}$.

4. Given $f(x) = \cos^2 x$, find $f^{(2k)}(0)$.

5. Find the arc length of the curve $y = \ln(1-x^2)$ from $x = 0$ to $x = 1/4$.

6. Let $g(x)$ be the inverse function of $f(x) = x^2e^x$ for $x \geq 0$. Find $\displaystyle\int_0^e g(x)dx$.

7. Find the tangent plane to the surface $\tan(xy) = \sin(yz)$ at the point $x = 1/4$, $y = \pi$ and $z = 1/2$.

8. Given $f(x,y) = x^3 + 2xy + y^3$, find the unit vector $\vec{u}$ such that that the directional derivative of $f(x,y)$ at the point $(1,2)$ and in the direction $\vec{u}$ attains its maximum.

9. Evaluate $\displaystyle\int_{x=0}^{x=8}\int_{y=x^{1/3}}^{y=2} \sin(y^4)dydx$.

10. Solve the differential equation $x^3y' = \sec y$, $y(1) = \pi/6$.

(A)、(B)兩題各 10 分，請寫出詳盡之計算與論證過程。

A. Evaluate $\displaystyle\int_{y=0}^{y=2/3}\int_{x=y}^{x=2-2y} (x+2y)e^{y-x}dxdy$ by making the change of variables $u = x + 2y$ and $v = y - x$.

B. Use Lagrange multipliers to find the minimal distance from the origin to the surface $z^2 = xy + 1$.

試題隨卷繳回