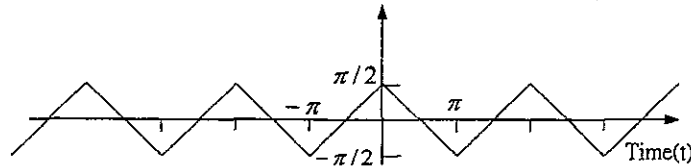


1. Find the solution of the following equation: (15%)

$$y'' + \lambda y = 0, \quad y(0) = y(L), \quad y'(0) = y'(L)$$

2. A time signal distribution is shown in the figure. Find the frequencies and the corresponding amplitudes contained in this signal. (15%)



3. (a) Find the solution of the following wave equation (10%)

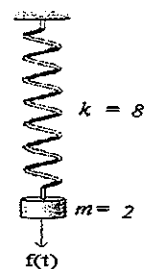
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty, \quad 0 < t < \infty, \quad ICs. \begin{cases} u(x, 0) = f(x) \\ u_t(x, 0) = g(x) \end{cases}$$

- (b) If $f(x) = 0$, $g(x) = x$, $0 \leq x \leq 1$, find the solutions of $u(-\frac{1}{2}, \frac{1}{3})$, $u(2, 5)$, and $u(\frac{1}{2}, \frac{1}{6})$ (5%)

4. Evaluate the line integral of $\oint_C \vec{v} \cdot d\vec{r}$, where $\vec{v} = xz\vec{j}$, and C is the trace of surface $z = 4 - y^2$, cut off by the planes $x = 0$, $z = 0$ and $y = x$. (15%)

5. Find the principal axes and transform the following equation $2x_1^2 + 4x_1x_2 + 5x_2^2 = 1$ to its canonical form. (10%)

6. Consider a mass and spring vibration system as shown in the figure. The non-dimensional parameters are shown in the figure. Assume that the system is initially at rest. And at time $t = 3$, a unit impulse force is applied downward on the system suddenly, find the displacement of the system, (10%)



7. Find the principal value of $\int_{-\infty}^{\infty} \frac{1}{(x^2 - 7x + 6)(x^2 + 4)} dx$ (10%)

8. A body of mass m is thrown vertically into the air with an initial velocity v_0 . If the body encounters an air resistance proportional to its velocity, find (a) the velocity of the body at any time t and (b) the time at which the body reaches its maximum height. (10%)