1. (35%)

(a) Consider the following primal dual pair:

$$(P) \qquad \min_{x} \ \mathbf{c}^T\mathbf{x}$$

$$\text{subject to} \quad \mathbf{Ax} \geq \mathbf{b},$$
$$\mathbf{x} \geq 0,$$

and

$$(D) \qquad \max_{y} \ \mathbf{b}^T\mathbf{y}$$

$$\text{subject to} \quad \mathbf{A}^T\mathbf{y} \leq \mathbf{c},$$
$$\mathbf{y} \geq 0.$$

Prove that if $\mathbf{x}$ is feasible for (P) and $\mathbf{y}$ is feasible for (D), then

$$\mathbf{c}^T\mathbf{x} \geq \mathbf{b}^T\mathbf{y}.$$

(b) Find the dual of

$$\min_{x,a} \ \mathbf{c}^T\mathbf{x}$$

$$\text{subject to} \quad \mathbf{Ax} = \mathbf{b},$$
$$\mathbf{x} \geq \mathbf{a},$$
$$\mathbf{a} \geq 0.$$

2. (15%) Solve the following nonlinear optimization problem (note: x_1 , $x_2 \in$ R)

$$\min_{x_1,x_2} \ x_1$$

$$\text{subject to} \quad x_1^3 \geq x_2,$$
$$x_2 \geq 0.$$

3. (25%) An airline company is considering the purchase of new long-range, medium-range, and short-range jet passenger airplanes. The purchase price would be \$35 million for each long-range plane, \$25 million for each medium-range plane, and \$17.5 million for each short-range plane. The board of directors has authorized a maximum commitment of \$750 million for these purchases. Regardless of which airplanes are purchased, air travel of all distances is expected to be sufficiently large that these planes would be utilized at essentially maximum capacity. It is estimated that the net annual profit (after capital recovery costs are subtracted) would be \$2 million per long-range plane, \$1.5 million per medium-range plane, and \$1.15 million per short-range plane. It is predicted that enough trained pilots will be available to the company to crew 30 new airplanes. If only short-range planes were purchased, the maintenance facilities would be able to handle 40 new planes. However, each medium-range planes is equivalent to 1.5 short-range planes, and each long-range plane is equivalent to 2 short-range planes in terms of their use of the maintenance facilities. Using the above information, management wishes to know how *many* planes of each type should be purchased to maximize profit.

(a) Formulate an IP model for the problem.

(b) Use a binary representation of the variables to reformulate the IP model in part (a) as a BIP problem and outline a methodology to solve this problem.

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4. (25%) A college student has 7 days remaining before final examinations begin in her four courses, and she wants to allocate this study time as effectively as possible. She needs at least 1 day on each course, and she likes to concentrate on just one course each day, so she wants allocate 1, 2, 3, or 4 days to each course. Having recently taken an operations research course, she decided to use dynamic programming to make these allocations to maximize the total grade points to be obtained from the four courses. She estimates that the alternative allocations for each course would yield the number grade points shown the following table:

	Estimated Grade Points			
	Courses			
Study Days	1	2	3	4
1	3	5	2	6
2	5	5	4	7
3	6	6	7	9
4	7	9	8	9

Solve this problem by dynamic programming.