Note: This is a close book and close note examination. You can use either Chinese or English to answer the questions below. The total number of points is 100. The number in parenthesis in front of each problem is the points for that problem. You need to show all the work to receive the points.

Take your time and GOOD LUCK.

(10) 1. Let X be a random variable of the continuous type that has pdf f(x). If m is the unique median of the distribution of X and b is a real constant, show that $E(|X-b|)$ is a minimum when b=m.

(20) 2. Let $Y_1 < Y_2$ denote the order statistics of a random sample of size 2 from $N(0, \sigma^2)$.

(a) Find $E(Y_1)$.

(b) Find the covariance of Y_1 and Y_2.

(20) 3. Let $\{X_n\}$ be a sequence of random variables and let X be a random variable. If X_n converges to X in probability, prove that X_n also converges to X in distribution.

(15) 4. The number of breakdowns per week for a certain type of main-frame computer is a random variable Y having the Poisson distribution with mean λ. A random sample $Y_1, Y_2, \ldots, Y_n$ of observations on the weekly number of breakdowns is available.

(a) Find an unbiased estimator of λ.

(b) The weekly cost for repairing these breakdowns is $C = 3Y + Y^2$. Evaluate E(C).

(c) Find a function of $Y_1, Y_2, \ldots, Y_n$ that is an unbiased estimator of E(C).

(10) 5. Suppose Y represent a single observation from the probability density function (pdf) given by

$$f(y|\theta) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the most powerful test with significance level $\alpha = 0.05$ to test
$H_o: \theta = 1$ vs. $H_a: \theta = 2$.

(25) 6. Let $Y_1, Y_2, \ldots, Y_n$ be independently and identically distributed as

$$f(y) = \begin{cases} \lambda e^{-\lambda y}, & y > 0 \text{ and } \lambda > 0 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the maximum likelihood estimator (MLE) of λ

(b) Is the MLE of λ in (a) unbiased?

(c) Is the MLE in (a) efficient for λ?

(d) Does an efficient estimator of λ exist?

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