國立臺灣大學95學年度碩士班招生考試試題

Problem 1. Consider matrix $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and column vector $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

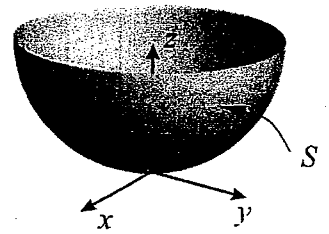
1) Find $\mathbf{y} = \mathbf{A}\mathbf{x}$. (5 %); **2)** Find $\mathbf{y} = \mathbf{A}^3\mathbf{x}$. (5 %); **3)** Find $\det(\mathbf{A}^9)$. (5 %)

4) Is matrix **A** an orthogonal matrix ? Show why or why not. (5 %)

5) Find a matrix **B** such that **x** is an eigenvector of **B**. (5 %)

Problem 2. With reference to the right figure, consider the surface *S* defined by

$$f(x,y,z) = x^2 + y^2 + (z-1)^2 = 1; \text{ and } \quad z \le 1$$



1) Find $\operatorname{grad} f = \nabla f$. (5 %)

2) Find unit normal vector $\mathbf{n} = (n_1, n_2, n_3)$ to surface *S* at position $(x,y,z) = (\frac{1}{3}, \frac{2}{3}, \frac{1}{3})$. (5 %)

3) Use Gauss's divergence theorem to determine the surface integral $\iint_S \mathbf{q} \cdot \mathbf{n}\, dA$

where **n** is the local unit normal to *S* and vector field **q** is defined as $\mathbf{q} = (q_1, q_2, q_3) = (2x, 0, z)$. (15 %)

Problem 3. Given a partial differential equation: $\frac{\partial u(x,y)}{\partial x} + 2\frac{\partial u(x,y)}{\partial x} = 2u(x,y) + 5\sin x$

1) Find the general solution of this PDE. (15%)

2) Provided that $u(x,0) = e^x$, find the exact solution of this PDE. (15%)

Problem 4. Given an ordinary differential equation: $y''(x) - y'(x) - 2y(x) = 36\cosh x$, with initial conditions $y(0) = 3$, $y'(0) = 0$, find the solution. Note: the complementary and the particular solutions must be individually specified. (20%)

(Note: Reasonable conditions can be assumed by the examinee, provided that the conditions provided are insufficient.)

試題隨卷繳回