Engineering Mathematics

1. (16%) Find the equation of the curve, the slope of which is $4 - 2x$, and which passes through the point $(2, 6)$.

2. (16%) Solve the differential equation:

$$(xy^2 - x)dx + (x^2y + y)dy = 0$$

with $y(x = 0) = \sqrt{2}$.

3. (16%) Use the Laplace transform to solve the following system of differential equations.

$$\begin{aligned} w' + y &= \sin(x) \\ y' - z &= e^x \\ w + y + z' &= 1 \end{aligned}$$

with $w(0) = 0$, $y(0) = 1$, $z(0) = 1$.

4. For an arbitrary $m \times n$ matrix A, its general inverse A^-, which has the dimension of $n \times m$, satisfies the following conditions: (i) Both AA^- and A^-A are symmetric, (ii) $A^-AA^- = A^-$, (iii) $AA^-A = A$. Find the general inverse of the following matrices:

(a) (8%) $F = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 5 \\ 2 \end{bmatrix}$,　　(b) (8%) $G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$

5. Check if each of the following vector fields is conservative. If it is conservative, find the corresponding potential function.

(a) (8%) $F(x, y, z) = [yze^{xyz} - 4x]\vec{i} + [xze^{xyz} + z + \cos(y)]\vec{j} + [xye^{xyz}]\vec{k}$

(b) (8%)
$G(x, y, z) = [2xyze^{x^2yz} - 2x + y]\vec{i} + [x^2ze^{x^2yz} + x]\vec{j} + [x^2ye^{x^2yz} - \sin(z)]\vec{k}$

6. (20%) Find the general solution of the following problem:

$$\frac{\partial^2 u(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial \theta^2} = 0, \quad 0 < r < 1,\ 0 < \theta < \alpha$$
$u(r, 0) = 0$, $u(r, \alpha) = 0$, and $u(0, \alpha)$ is bounded.

試題隨卷繳回