國立臺灣大學95學年度碩士班招生考試試題

(1) Find the eigen values and their corresponding eigen vectors of the following problem:
$\mathbf{Ax} = \lambda \mathbf{x}$

$$\text{where} \quad \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

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(2) Consider a problem of 3rd order O.D.E of $y(x)$: $y''' + y'' + y = \cos x$, if the initial conditions are given as: $y(0) = A; y'(0) = B; y''(0) = C$ where $A; B; C$ are constants. This problem can be rewritten by a system of three simultaneous 1st order O.D.E. of the form:

$$y_1' = f_1(y_1, y_2, y_3, x)$$
$$y_2' = f_2(y_1, y_2, y_3, x),$$
$$y_3' = f_3(y_1, y_2, y_3, x)$$

Find $f_1; f_2; f_3$ and the initial conditions of $y_1(x); y_2(x); y_3(x)$

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(3) Find the Fourier series expansion of the function $f(x)$: $f(x) = (1 + 2\sin 2x)^2$

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(4) The general solution of the 1-D free space wave equation of $u(x,t)$:

$(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2})u = 0$, is given by $u(x,t) = f(x-ct) + g(x+ct)$, where f and g are arbitrary functions of $(x-ct)$ and $(x+ct)$ respectively. Find the general solution of the following equation of $u(r,t)$:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial u}{\partial r}) = 0$$

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(5) Use the method of Laplace Transform, solve the following problem of $y(t)$:

$$\frac{d^2}{dt^2}y + 2\frac{d}{dt}y + 2y = \delta(t) \quad ; \quad y(0) = \dot{y}(0) = 0$$

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(6) Determine an analytic complex function $f(z)$, where $z = x + iy$, such that $f(z) = u + iv$ and $u(x,y) = y^3 - 3x^2 y + y$.

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