1. Please find a (real) general solution for following differential equations. (Show all details of your calculations)

(1) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = e^{2x}\csc x$ (15%)

(2) $x\dfrac{dy}{dx} + 3y = x^2 \sin x$ (10%)

2. One of following differential equation systems is used to model the competing species in biology.

(1) Please select and solve a suitable system from differential equations (a) and (b) for modeling the simple competing species and obtain its trajectories. (10%)

Models:

$$\frac{dx}{dt} = 0.2x - 0.02xy, \quad \frac{dy}{dt} = 0.02xy - 1.2y \qquad \text{(a)};$$

$$\frac{dx}{dt} = 2x - 0.3xy, \quad \frac{dy}{dt} = 4y - 0.7xy \qquad \text{(b)}$$

(2) Please find critical points with their eigenvalues for the model. (10%)

3.Please solve the following system by only using the Gauss-Jordan elimination, and then indicate the existence of solutions or no solutions by using Rank of matrix. (Show the details and reasons of your work and solutions) (15%)

$$\begin{aligned} -6x_1 + x_2 - 4x_3 &= 1 \\ 2x_1 - x_2 - x_3 &= 8 \\ x_1 + 6x_2 - x_3 &= -3 \end{aligned}$$

4. Please describe the region of integration $\int_0^3 \int_{-y}^{y} (x^2 + y^2)\,dx\,dy$ and evaluate it. (10%)

5. Please solve the following boundary value problem using separation of variables. (30%)

$$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2} \quad \text{for} \quad 0 < x < 3, \quad t > 0,$$

$$y(0,t) = y(3,t) = 0 \quad \text{for} \quad t \ge 0,$$

$$y(x,0) = 0, \quad \frac{\partial y}{\partial t}(x,0) = x(3-x) \quad \text{for} \quad 0 \le x \le 3.$$