

1. (15 points) Given a number sequence $S[1..n]$, the MaxSum procedure outputs

$\max\{0, \max_{1 \leq i \leq j \leq n} \sum_{k=i}^j S[k]\}$. Complete the following pseudocode.

```

MaxSum (S, n)
begin
  x ← 0; y ← 0
  for i ← 1 to n
    do if  $S[i] + y > x$  then y ← 1A; x ← 1B
       elseif  $S[i] + y > 0$  then y ← 1C
       else y ← 0
  print x
end

```

2. (15 points) Complete the following pseudocode so that it prints "123 123 132 123 123 213 213 231 213 123 321 321 312 321 123" in order when it is invoked by $X(S, 1, 3)$, where $S[i]=i$ for $1 \leq i \leq 3$.

```

X(S, k, n)
begin
  if  $k=n$  then print  $S[1..n]$ 
  else
    for i ← k to n
      do temp ← S[k]
         S[k] ← S[i]
         S[i] ← temp
         X(S, 2A, 2B)
         temp ← S[k]
         S[k] ← S[i]
         S[i] ← temp
         X(S, n, n)
    end
end

```

3. (20 points) **The binary-search-tree property:** Let x be a node in a binary search tree. If y is a node in the left subtree of x , then $\text{key}[y] \leq \text{key}[x]$. If y is node in the right subtree of x , then $\text{key}[x] \leq \text{key}[y]$. Suppose that we have integer numbers between 1 and 10000 in a binary search tree and want to search for the number 2006. The following sequence is the sequence of nodes examined. Give all the feasible ranges of the variable k in the sequence.

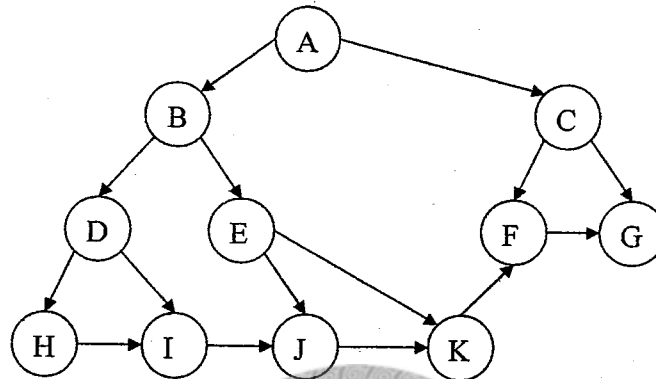
1000, 5566, 5203, k , 1314, 1510, 2381, 2006

4. (20 points) Give tight asymptotic bounds for the following recurrences.

- $T(n) = 2T(\frac{n}{2}) + n$
- $T(n) = T(\frac{n}{4}) + T(\frac{3n}{4}) + n$
- $T(n) = T(\frac{n}{7}) + T(\frac{3n}{4}) + n$
- $T(n) = 7T(\frac{n}{4}) + n^2$

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5. (15 points) Add one edge to the following directed graph to make it strongly connected.



6. (15 points) Complete the following Depth-First Search pseudocode, which is used to classify the edges of the input graph $G=(V, E)$:

DFS(G)

1. for each vertex $u \in V[G]$
2. do $\text{color}[u] \leftarrow \text{WHITE}$
3. $\pi[u] \leftarrow \text{NIL}$
4. $\text{time} \leftarrow 0$
5. for each vertex $u \in V[G]$
6. do if $\text{color}[u] = \text{WHITE}$
7. then DFS-VISIT(u)

DFS-VISIT(u)

1. $\text{color}[u] \leftarrow \text{GRAY}$
2. $\text{time} \leftarrow \text{time} + 1$
3. $d[u] \leftarrow \text{time}$
4. for each $v \in \text{Adj}[u]$
5. do if $\text{color}[v] = \text{WHITE}$
6. then $\pi[v] \leftarrow u$
7. DFS-VISIT(v)
8. (u, v) is a tree edge
9. elseif $\text{color}[v] = \text{GRAY}$
10. then (u, v) is a 6A edge
11. elseif $\text{color}[v] = \text{BLACK}$
12. then if 6B
13. (u, v) is a cross edge
14. else (u, v) is a 6C edge
15. $\text{color}[u] \leftarrow \text{BLACK}$
16. $\text{time} \leftarrow \text{time} + 1$
17. $f[u] \leftarrow \text{time}$