

1. Can the photoelectric effect occur for completely free electrons? Prove your statement in details. (15 points)

2. A particle of mass  $\mu$  moves in the **3-dimensional** potential  $V(r) = Ar^2 + \frac{B}{r^2}$ , where A, B > 0. Find the energy levels. (15 points)

3. The wave function for an electron in a certain one-dimensional potential is  $\psi_0(x, t) = A_0 \exp(-\pi m \omega x^2 / \hbar - i \omega t / 2)$  where  $A_0$  is a real number. (a) Find  $A_0$  assuming the wavefunction is normalized. (10 points) (b) Where on the x axis is the electron most likely to be found? (Hint:  $\int_0^\infty \exp(-ax^2) = \frac{1}{2} \sqrt{\frac{\pi}{a}}$ ) (10 points)

4. Describe briefly about the production mechanism for X-rays and draw a typical X-ray spectrum (5 points). Lay out the apparatus needed to measure the X-ray spectra (5 points). What is the Bragg's law (5 points)? X-rays can scatter off an atom on a crystal lattice in all directions. Explain why, in deriving the Bragg's law, we only consider the reflection direction from a crystal plane/surface for an incident X-ray (5 points).

5. Isospin is a symmetry introduced by Heisenberg to explain some properties of nuclear interactions involving proton (p), neutron (n) and pion ( $\pi$ ). Proton is assigned to have isospin  $I=1/2$  with the z projection component  $I_z=1/2$  (i.e.  $|1/2, 1/2\rangle$  or isospin up state); neutron is assigned to the  $|1/2, -1/2\rangle$  state. What will be the possible isospin state for deuteron (5 points)? You can copy the idea for the addition of the angular momentum here to the case of isospin since they behave the same mathematically. If you know that  $\Delta^+$  is assigned to the isospin  $|3/2, 1/2\rangle$  state and  $\Delta^+$  can decay either to  $p\pi^0$  or to  $n\pi^+$ , what will be the decay percentage for each mode assuming isospin is a good symmetry (10 points)? Note that  $\pi^+$  is an isospin  $|1, 1\rangle$  state and  $\pi^0$  is  $|1, 0\rangle$  state; the ladder operator for spin has the following relationship in Natural units:  $S_-|sm\rangle = \sqrt{[s(s+1)-m(m-1)]} |s(m-1)\rangle$ .

6. One plans a nuclear experiment with a high speed proton bombarding at a lithium nucleus ( $\text{Li}^7$ ) at rest. If the nuclear interaction can only happen with a distance about 1 fm, what will be the kinetic energy (in eV) needed for the incident proton to overcome the Coulomb barrier (5 points)? Historically people observed two energetic helium nuclei (about 8 MeV each) coming out of this reaction and the accelerating potential for proton is only about 200 kV. Write down this nuclear reaction and explain why this can happen (10 points).