1. (20 %)
   (a) Find the Fourier series expansion, $S_F$, for $f(x) = -x,\ -1 < x < 1$.
   (b) Accordingly, find the Fourier series expansion, $S_h$, for $h(x) = 2 - x,\ 2 < x < 6$.

2. (20 %)
   For a vector function $\underline{F}(x,\ y,\ z) = F_1\underline{i} + F_2\underline{j} + F_3\underline{k}$, it is known that

$$curl\underline{F} = \nabla \times \underline{F} = (-4y^3z^6 - 4x^6y^2)\underline{i} - 4z^3\underline{j} + (20x^4y^2z - 3x^2y^2)\underline{k},$$
$$div\underline{F} = \nabla \bullet \underline{F} = 2xy^3 + 8x^5yz - 6y^4z^5.$$

   Find the possible $F_1$, $F_2$ and $F_3$. (Hint: There is no unique solution. The solution based on observation is recommended.)

3. (25 %)
   The definition of Laplace transform is

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt.$$

   (a) Derive $L[\delta(t-c)] = e^{-sc}$, where $\delta(t)$ is the Dirac delta function.
   (b) Show $L[\frac{df(t)}{dt}] = sL[f(t)] - f(0)$, provided that $f(\infty) = 0$.
   (c) Show $L[f(t-a)H(t-a)] = e^{-as}L[f(t)]$, where $H(t)$ is the Heaviside step function.
   (d) Solve

$$\frac{d^2m(t)}{dt^2} = \delta(t-75),\quad 0 \le t < \infty,$$

   with

$$m(0) = m(\infty) = 0.$$

4. (10 %)
   The solution of the second order ordinary differential equation $y''(x) - 2y'(x) + y(x) = 0$ can be written as $y(x) = C_1y_1(x) + C_2y_2(x)$. If given $y_1(x) = e^x$, then derive the second solution as $y_2(x) = xe^x$.
   Note: no derivation, no score!

5. (25 %)
   For the second order ordinary differential equation

$$(2x+3)^2y''(x) - 2(2x+3)y'(x) + 4y(x) = 0,$$

   (a) derive and obtain the transformation $t = f(x)$ which transforms this differential equation to a constant coefficients second order ordinary differential equation of variable $t$ (Hint: chain rule).
   (b) Write the transformed second order ordinary differential equation of variable $t$.
   (c) Solve $y(x) = ?$

試題隨卷繳回