1. **(10%)** Let two vectors $\mathbf{P}=(2,2,1)$ and $\mathbf{Q}=(1,-2,0)$. Calculate (i) $\mathbf{P}\cdot\mathbf{Q}$ and (ii) $\mathbf{P}\times\mathbf{Q}$.

2. **(10%)** Prove that for any three vectors **P**, **Q**, and **R**, $(\mathbf{P}\times\mathbf{Q})\times\mathbf{R}=(\mathbf{P}\cdot\mathbf{R})\mathbf{Q}-(\mathbf{Q}\cdot\mathbf{R})\mathbf{P}$

3. **(15%)** Orthogonalize the following set of vectors.

$$\mathbf{e}_1=(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0)$$
$$\mathbf{e}_2=(-1,1,-1)$$
$$\mathbf{e}_3=(0,-2,-2)$$

4. **(15%)** Calculate the eigenvalues and eigenvectors of the matrix $\mathbf{M}=\begin{bmatrix}1 & 1\\ 3 & -1\end{bmatrix}$.

5. **(15%)** Calculate the inverse of the following matrix $\mathbf{M}=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$.

6. **(10%)** Expand $1/x$ in a Taylor series to second order about $x=1$.

7. **(10%)** Find the Jacobian matrix $\begin{bmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{bmatrix}$ for the mapping $x=\cos u\cos v$, $y=\cos u\sin v$.

8. **(15%)** Find the general solution for the force vibration example $\frac{d^2u}{dt^2}+\omega_o^2u=F_0\cos\omega t$.

試題隨卷繳回