1. Derive the conservation equations for linear momentum and angular momentum using the Reynolds transport theorem. (15 %)

2. Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Figure 1. The exit area of each of the two nozzles is 30 mm^2, and the flow leaving each nozzle is in the tangential direction. The radius from the axis of rotation to the centerline of each nozzle is 200 mm. (a) Determine the resisting torque required to hold the sprinkler head stationary. (b) Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. (c) Determine the speed of the sprinkler if no resisting torque is applied. (15%)

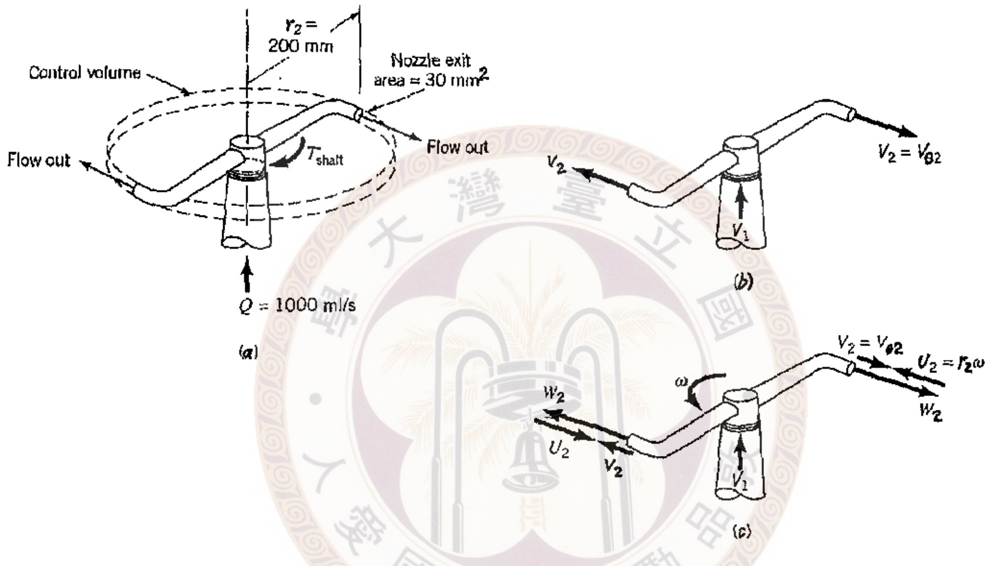

Fig. 1

3. If you are given velocity potentials (ϕ) and stream functions (ψ) as follows:
   i) Uniform flow at angle α with the x axis: $\phi = U(x\cos\alpha + y\sin\alpha)$, $\psi = U(y\cos\alpha - x\sin\alpha)$.
   ii) Doublet: $\phi = K\cos\theta / r$, $\psi = -K\sin\theta / r$, where K is the strength.
   iii) Free vortex: $\phi = \Gamma\theta / (2\pi)$, $\psi = -\Gamma \ln r / (2\pi)$, where Γ is the circulation.

(a) Derive the velocity distribution around a circular cylinder with the assumption of negligible viscosity. (If using the potential flow theory, you may assume $\psi = 0$ for $r = a$, the radius of the cylinder) What are the resultant forces (per unit length) developed on the cylinder respectively in x and y directions? (10 %)
(b) If the cylinder is rotating, what is the velocity distribution on the surface? By calculating the net force exerted on the cylinder, thus explain the motion of a baseball when it spins. (10 %)

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4. An incompressible fluid oscillates harmonically $(V = V_0 \sin \omega t)$ with a frequency of 10 rad/s in a 4-in.-diameter pipe. A $\frac{1}{4}$ scale model is to be used to determine the pressure difference per unit length, Δp_l (at any instant) along the pipe. Assume that $\Delta p_l = f(D, V_0, \omega, t, \mu, \rho)$ where D is the pipe diameter, ω the frequency, t the time, μ the fluid viscosity, and ρ the fluid density. (25%)

   (a) determine the similarity requirements for the model and the prediction equation for Δp_l.

   (b) If the same fluid is used in the model and the prototype, at what frequency should the model operate.

5. A circular water pipe has an abrupt expansion from diameter $D_1 = 15cm$ to $D_2 = 20cm$.(Fig 2) The pressure and the average water velocity in the smaller pipe are $p_1 = 120kPa$ and $10m/s$, respectively, and the flow is turbulent. By applying the continuity, momentum, and energy equations and disregarding the effects of the kinetic energy and momentum flux correction factors, show that the loss coefficient for sudden expansion is $K_L = (1 - D_1^2 / D_2^2)^2$, and calculate K_L and P_2 for the given case. The density of water is taken to be $\rho = 1000kg/m^3$. (15%)

6. For each statement, choose whether the statement is true or false and discuss your answer briefly. These statements concern a laminar boundary layer on a flat plate. (Fig.3) (10%)

   (a) At a given x-location, if the Reynolds number were to increase, the boundary layer thickness would also increase.

   (b) As outer flow velocity increases, so does the boundary layer thickness.

   (c) As the fluid viscosity increases, so does the boundary layer thickness.

   (d) As the fluid density increases, so does the boundary layer thickness.

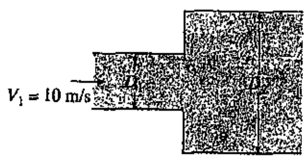

Fig. 2

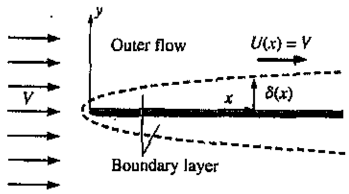

Fig. 3