總分 100 分，請依序作答，並標明題號

1. (15%)

Consider the inverted pendulum system as shown in Fig.1. Assume that the mass of the inverted pendulum is m and the center of gravity of the pendulum is located at the center of the rod. Assume that θ is small.

(a) Please derive the mathematical models in the form of differential equations. **(5%)**

(b) Please derive the mathematical models in the form of transfer functions. **(5%)**

(c) Please derive the mathematical models in the form of state-space equations. **(5%)**

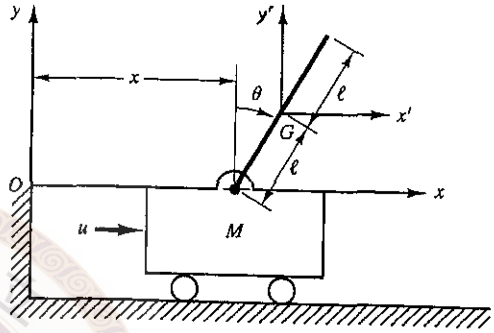

Fig. 1

2. (20%)

Consider the system shown in Fig. 2, which involves velocity feedback. Determine the values of the amplifier gain K and the velocity feedback gain K_h so that the following specifications are satisfied:

(a) Damping ratio of the closed-loop poles is 0.5

(b) Settling time ≤ 2 sec

(c) Static velocity error constant $K_V \geq 50 \text{ sec}^{-1}$

(d) $0 < K_h < 1$

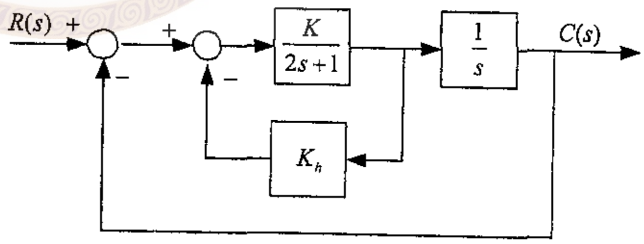

Fig.2

3. (20%)

Consider the unity-feedback system which open loop transfer function is $G(s) = \dfrac{K}{s(s+1)^2}$.

(a) Determine the range of K for system stability by Nyquist stability criterion **(10%)**

(b) Determine the phase margin and the gain margin as K=0.5 **(10%)**

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4. (25%)

A control system, as shown in Fig. 3, has the control target $y_{set} = 0$, the output $y(t)$ and the disturbance $d(t)$.

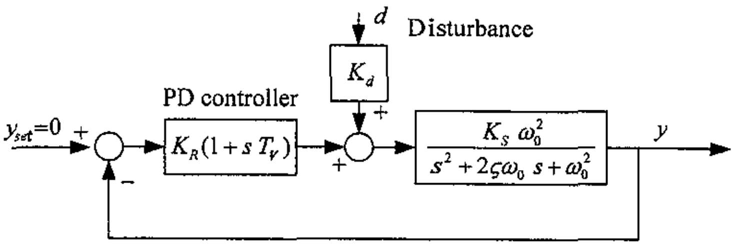

Fig. 3

(a) Derive the transfer function $G_D(s)=\frac{Y(s)}{D(s)}$ for the disturbance $d(t)$, and solve the output response $y(t)$ as $d(t)=\delta(t)$, where $\delta(t)$ is an impulse function as $\delta(t)=\begin{cases}\infty & for\ t=0\\ 0 & for\ t\neq 0\end{cases}$ (7%)

(b) Solve the output response $y(t)$ as $d(t)=D_0\cos\omega_0 t$ with zero initial values (8%)

(c) Solve the PD-controller, K_R and T_V, for keeping the damping ratio $\varsigma_C = 0.707$ and the natural frequency $\omega_{0C} = 10\,s^{-1}$ in the close-loop system, where

$\varsigma = 0.1;\ \omega_0 = 2\,s^{-1};\ K_S = 3;\ K_d = \frac{1}{3}$. (10%)

5. (20%)

Consider the control system as $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}+\mathbf{B}u,\ y=\mathbf{C}\mathbf{x}+\mathbf{D}u$, where

$$\mathbf{A}=\begin{bmatrix}1 & -1 & -1\\ -1 & -1 & -1\\ 1 & 0 & -1\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}2\\ 0\\ -1\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}1 & 0 & 1\end{bmatrix},\ \mathbf{D}=\begin{bmatrix}0\end{bmatrix}$$

The system uses the state feedback control $u=-\mathbf{K}\mathbf{x}$. Please solve the controller $\mathbf{K}$ for the desired closed-loop poles at s=-5, -10, -15.